TFEE-1 (Practical)

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

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Guidelines to independent and practical works on discipline THEORETICAL FUNDAMENTALS OF ELECTRICAL ENGINEERING

For full-time students' majoring in 141 "Electric Power, Electrical Engineering and Electromechanical"

Part 1
THEORY FUNDAMENTALS OF DC AND SINGLE-PHASE HARMONIC AC CIRCUITS

Dnipro
2021

Рекомендовано до видання навчально-методичним відділом (протокол № від за поданням науково-методичної комісії зі спеціальності 141 – Електроенергетика, електротехніка та електромеханіка (протокол № 21/22-01 від 30.08.2021 р.)

Методичні вказівки до самостійних та практичних занять і контрольні завдання з дисципліни «Теоретичні основи електротехніки, частина 1 (розділи: «Основи теорії кіл постійного струму» та «Основи теорії кіл гармонійного однофазного струму») для студентів спеціальності 141 – Електроенергетика, електротехніка та електромеханіка /В.С. Хілов; Нац. техн. ун-т. «Дніпровська політехніка» – Д.: НТУ "ДП", 2021. – 44 с.

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Методичні вказівки призначено для виконання самостійної роботи і контрольних завдань та проведення практичних занять з дисципліни "Теоретичні основи електротехніки" (частина 1, розділи "Основи теорії кіл постійного і гармонійного однофазного струмів") студентами денної та заочної форм навчання за спеціальностями: 141 Електроенергетика, електротехніка та електромеханіка.

У кожному розділі подано короткі методичні вказівки, типові завдання з рішенням та необхідними поясненнями, а також вихідні дані для виконання самостійно студентами розрахунково-графічних завдань. Наводяться питання для самостійного контролю залишкових знань.

Друкується в редакції автора.

GENERAL METHODOLOGICAL INSTRUCTIONS K CALCULATED-GRAPHICAL TASKS

In the studies of the theory of electric circuits main attention follows to pay, to as ability to compose the algorithm of calculation on the grounds of theoretical positions, so I to analysis arising in them physical processes. Neglect either of these positions inadmissible, since entail ununderstanding of arising phenomena in electric chains.

After learning theory is recommended investigate in deciding of tasks as to this topic (see literature), then independent to decide of a few simple tasks. The purpose of calculated-graphical tasks - final review learning by the students of fitting chapters of course, Follows emphasize, what theory of processes arising in electric circuits or electrotechnical devices, is enounced on mathematical basis, that is why from student is needed ability free to use mathematical apparatus.

Schemes, pictures, graphs to fulfill according to the demands of the common system of design documentation, hereto graphs - on linear graph paper with indication not the axes of the coordinates of values and the unit of their measurement. In calculations to lead necessary computative formulae. Results to distinguish from scorching text. Fulfilling decisions (every stage must be explained in brief), do not do all algebraic manipulations, but only main ones. In calculations (accurate within third signifying figures) to use calculator or PC, In computative-graphic tasks (the sample of title see in annex) had in the doctrines of formula and equations do not carry out. To end work by list of references, by the date of its end and by signature.

Size of calculated-graphical task is established by tutor.

Calculated-graphical tasks included in these methodological instructions, do not embrace all program. That's why the separate chapters of course necessary to master independent rememberring, what each of them is is important equally. It is it is impossible to embark on the studies of subsequent chapters, having not internalized previous.

The number of fulfiled variant is chosen according to the course for credit book of student and correspond to two last numbers one.

1. BASIC THEORY OF THE DC ELECTRIC CIRCUIT

The reactance elements do not show their own internal in DC circuits in static regimes. In this regimes it is necessary to allow resistance elements only. Therefore we are beginning electrical circuits study regularities in static regimes DC circuits.

We commence our study by defining some basic concepts. Before defining these concepts, we must first establish a system of units that we will use throughout the text, see Appendix A.

1. Linear DC Circuits Parameters Calculation

1.1. Analysis Methods of DC circuits

When analyzing electric DC circuits with active resistances it is necessary to take into account the following:

1. All variables are the set of real numbers with the sign, which are located on the single number line. The calculation is based on algebraic equations using matrix apparatus for the solutions of system equations.
2. The current of ideal current source is not dependent on the load resistance. The inner resistance of ideal current source is always equal to infinity.
3. The output voltage of ideal source of EMF is not dependent on load current. The internal resistance of ideal source of EMF is always equal to zero.
4. In the steady-state modes of DC electric circuits, only the ohmic resistances of elements are taken into account in the calculation. The perfect inductive elements do not have ohmic resistance, and perfect capacitive elements disconnected the branches in DC circuits because they have infinitely large internal ohmic (active) resistance.
5. The calculation of linear DC circuits in steady-state operation is performed on the basis of algebraic equations, which are drawn up using Kirchhoff's laws, and in some cases, when the circuit has the only source of energy – on the basis of Ohm's law.
6. In the cases, when the quantity of independent loops is less than the number of branches then it is rational to perform circuit calculation on the basis of the mesh currents method.
7. If the number of nodes in circuit is less than the number of independent loops, then it is advisable to do currents calculation on the basis of the nodal potentials method, and in particular case when circuit contain only two nodes – on the basis of the method of two nodes.
8. When it is necessary to calculate the parameters of circuit only in single branch then it is rational to perform calculation on the basis of the equivalent generator method.
9. The verification of the calculation correctness is performed by means of drawing up the power balance.
10. When the power balance is drawn up it is necessary to take into account that ohmic resistance elements are always consuming electric energy, and it is irreversibly converted into heat.

11. Energy sources can both consume and deliver electric energy. If the voltage and current at the terminals of the energy source have the same direction, then source consumes energy, otherwise source delivers it.

12. Transformation of the current source to the source of EMF and inverse transformation may change the operation mode of initial and converted power source.

13. Potential circle is the graph of the voltage distribution along any part of circuit. If the circuit part is not closed, then it is graphic representation of Ohm's law, otherwise it is graphic representation of second Kirchhoff's law.

1.2. Ohm's Law Using to Calculate DC Circuit Parameters

Ohm's law is established the relationship between current, voltage and resistance of a branch in unbranched circuits i.e. the current is directly proportional to the potential difference at the terminals of a circuit section and inversely proportional to the resistance of this on. To write Ohm's law in mathematical form, a positive current direction is chosen arbitrarily. So, for branches without electromotive forces (EMF) (Fig.1.1: for example, for parameters $E_2 = 10V; E_3 = 20V;$ $R_1 = 10\Omega; R_2 = 15\Omega; R_3 = 20\Omega;$ $R_6 = 10\Omega;$ $R_4 = 10\Omega;$ $R_5 = 5\Omega$) we can choose positive directions of currents:

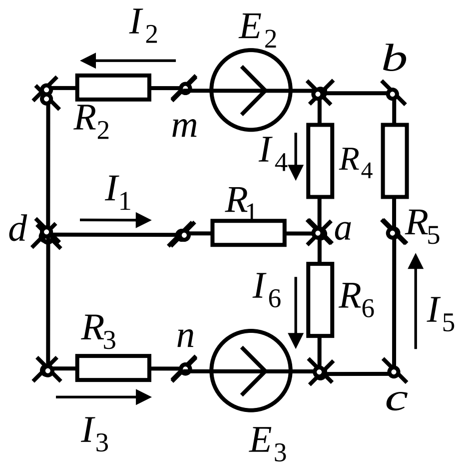


Fig.1.1

– I_1 direction from *d* to *a;*
– I_4 direction from *b* to *a;*
– I_6 direction from *a* to *c.*

The currents numerical values can be obtained by directly applying Ohm's law to the circuit branches

$$I_1 = \frac{V_d - V_a}{R_1} = \frac{0 - 6.09}{10} = -0.609, A;$$

$$I_4 = \frac{V_b - V_a}{R_4} = \frac{8.7 - 6.09}{10} = 0.261, A;$$

$$I_6 = \frac{V_a - V_c}{R_6} = \frac{6.09 - 9.57}{10} = -0.348, A,$$

where V_a, V_b, V_c, V_d are the known potentials of points *a*, *b*, *c* and *d* respectively ($V_c = 9.57V; V_d = 0.0V\ V_a = 6.09V; V_b = 8.7V$); $U_{da} = V_d - V_a$, $U_{ba} = V_b - V_a$, $U_{ac} = V_a - V_c$ are the voltages drop across sections between points *d* and *a, b* and *a, a* and *c.*

For the circuit sections with EMF sources and active resistances, Ohm's law is converted to the form:

$$I = \frac{V_1 - V_2 \pm E}{R}, A.$$

When using Ohm's law on the circuit section with the EMF source, we take into account the rule of signs for the EMF. The EMF sign is chosen positive “+” when the directions of the EMF and current in the circuit section are the same in direction (coincide), and the negative “-” sign is chosen if their directions are opposite (do not coincide). So, for the circuit sections (Figure 1.1) with EMF sources we have:

$$I_2 = \frac{V_b - V_d + E_2}{R_2} = \frac{8.7 - 0 + 10}{15} = 1.25, A;$$
$$I_3 = \frac{V_d - V_c - E_3}{R_3} = \frac{0 - 9.57 - 20}{20} = -1.48, A.$$

1.3. Kirchhoff's Laws Using to Calculate the Parameters of DC Circuit

Kirchhoff’s Currents Law (KCL). The first Kirchhoff's law is called the “currents law”. The current arriving at any junction point in a circuit is equal to the current leaving that junction, or the number of electric charges entering any node of the circuit is equal to the number of electric charges leaving this node, i.e. the algebraic sum of currents converging at the node is equal to zero

$$\sum_{k=1}^{n} I_k = 0,$$

where *k* is the running number of the branch connected to the node; in total *n* branches are connected to the node.

Kirchhoff’s Voltages Law (KVL). The second Kirchhoff's law states that the algebraic sum of voltage drops in any closed circuit (loop) is equal to the algebraic sum of the energy sources EMFs connected in this loop

$$\sum_{k=1}^{n} R_k I_k = \sum_{k=1}^{n} E_k,$$

where *k* is the running number of the branch that connected to the loop; in the loop are connected *n* branches.

The procedure to calculating the parameters of the circuit according to Kirchhoff's laws.

This method is used to calculate complicated branched circuits with several sources of electrical energy. The main idea in using these laws is to find joint equations, from the solution of which the sought parameters of the circuit are found. When draw up a system of equations, the following rules are used:

- The number of unknown quantities that need to be calculated is determined;

- The positive directions of unknown currents and voltages in the circuit are selected;
- The number of nodes, branches and independent loops in the circuit is counted;
- Joint equations are draw up according to the first Kirchhoff's law, the number of which is one less than the number of nodes;
- Missing equations are draw up using Kirchhoff's second law for independent loops. The total number of equations must be equal to the number of unknown parameters.

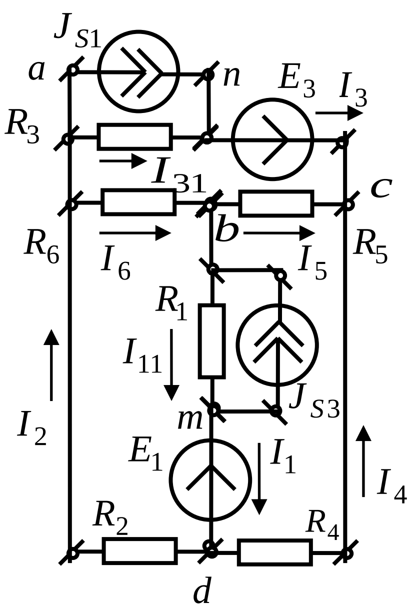


Fig.1.2

Calculating example # 1: Determine the branch currents in the circuit Fig.1.2 using Kirchhoff's laws, if known J_{S1}=1 A; J_{S3}=0.5 A; $E_1 = 10V; E_3 = 20V;$
$R_1 = 10\Omega; R_2 = 15\Omega; R_3 = 20\Omega;$
$R_6 = 10\Omega;\ R_4 = 10\Omega; R_5 = 5\Omega.$

Solution of the calculating example # 1:
This circuit contains six nodes and ten branches. We know the currents in the first and third branches J_{S1}=1 A, J_{S1}=0.5 A. It is necessary to determine eight currents. We choose arbitrary the positive directions for the currents $I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8$ and indicate them in the circuit. We can draw up five linearly independent equations according to the first Kirchhoff's law

for node “a” $-J_{S3}+I_{31}-I_6-I_2=0$ (1)
for “b” node $I_6-I_5+J_{S1}-I_{11}=0$ (2)
for “c” node $I_3+I_4-I_5=0$ (3)
for “d” node $I_1-I_2-I4=0$ (4)
for “n” node $J_{S3}+I_{31}-I_3=0$ (5)

According to the KVL (second Kirchhoff's law), we write three equations, choosing the direction of traversing the loops clockwise:

for “ancba” loop $I_{31}R_3-I_5R_5-I_6R_6=E_3$ (6)
for “abmda” loop $I_6R_6+I_{11}R_1+I_2R_2=-E_1$ (7)
for “bcdmb” loop $I_5R_5-I_4R_4-I_{11}R_1=E_1$ (8)

We write down the system of equations (1) ... (8), which contains six unknown currents:

$$-0.5+I_{31}-I_6+I_2=0$$
$$I_6-I_5+1-I_{11}=0$$

$$I_3+I_4-I_5=0$$
$$I_1-I_2-I4=0$$
$$0.5+I_{31}-I_3=0$$
$$I_{31}20-I_5 5-I_6\,10=20$$
$$I_6\,10+I_{11}10+I_2 15=-10$$
$$I_5 5-I_4\,10-I_{11}10=10$$

The solution of the equations system gives the numerical result:

$$I_1=-1.0\ A;\ I_{11}=0.0\ A;\ I_2=-0.0\ A;\ I_3=1.0\ A;\ I_{31}=0.5\ A;\ I_4=-1.0\ A;$$
$$I_5=0.0\ A;\ I_6=-1.0\ A.$$

1.4. Nodal Potentials Method Using for Calculating Parameters of DC circuit. Checking Calculation by Powers Balance

The nodal potentials method is the most common method for analyzing electrical circuits. This method is based on determining the potentials of each node as an independent variable. If the circuit all nodes potentials are determined, then according to Ohm's law, the current through each branch can be determined. In accordance with Ohm's law, the branch current is determined as follows:

$$I=(E+(V_1-V_2))/R$$

Each node current can be determined through the values of the nodal potentials, EMFs, branch conductivity and in accordance with the first Kirchhoff's law (KCL):

$$\sum I=0$$

The procedure for calculating the method of nodal potentials as applied to linear electrical circuits:

1. In the electrical circuit the number of all nodes are counted and the number of all nodes as *n* is denoted.
2. Select one reference node (this one usually is grounded). All other nodes potentials will be referenced to this node.
3. Using the first Kirchhoff's law to each of the n-1 nodes, we express each node current through the values of the node potentials and the conductance of the branches

$$V_1G_{11}-V_2G_{12}-V_3G_{13}-\ldots-V_{n-1}G_{1n-1}=I_{11}$$
$$-V_1G_{21}+V_2G_{22}-V_3G_{23}-\ldots-V_{n-1}G_{2n-1}=I_{22}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$-V_1G_{n-11}-V_2G_{n-12}-V_3G_{n-13}-\ldots+V_{n-1}G_{n-1n-1}=I_{n-1n-1}$$

4. Solving a linear system of *n-1* equations with *n-1* unknown potentials V_1, V_2, V_3, …, V_{n-1}
5. The branch currents are calculated according to Ohm's law.

Calculating example # 2: Determine the currents through the branches in the electrical circuit Fig.1.3, if known

J_{S2}=1 A; J_{S3}=0.5 A; $E_2 = 10V; E_3 = 20V;$

$R_1 = 10\Omega; R_2 = 15\Omega; R_3 = 20\Omega;$

$R_6 = 10\Omega; R_4 = 10\Omega; R_5 = 5\Omega.$

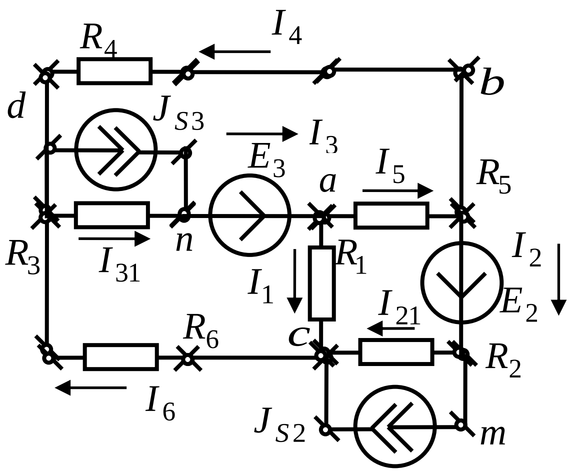


Fig.1.3

Solution of the Calculating example # 2: This circuit has 5 nodes, which are denoted as *a, b, c, d, n, m.* We ground the node *m* and determine its potential as V_m=0. Since the internal resistance of ideal source EMF is zero therefore $V_b = -E_2$.
For the remaining nodes *a, c,d* and *n* one can write the following equations:

$$GaaVa - GabVb - GacVc - GadVd - GanVn = Iaa$$
$$-GbaVa + GbbVb - GbcVc - GbdVd - GbnVn = Ibb$$
$$-GcaVa - GcbVb + GccVc - GcdVd - GcnVn = Icc$$
$$-GdaVa - GdbVb - GdcVc + GddVd - GdnVn = Idd$$
$$-GnaVa - GnbVb - GncVc - GndVd + GnnVn = Inn$$

Since the potential of point *b* (node *b*) is known, then

$$GaaVa - GacVc - GadVd - GanVn = Iaa + GabVb$$
$$-GbaVa - GbcVc - GbdVd - GbnVn = Ibb - GbbVb$$
$$-GcaVa + GccVc - GcdVd - GcnVn = Icc + GcbVb$$
$$-GdaVa - GdcVc + GddVd - GdnVn = Idd + GdbVb$$
$$-GnaVa - GncVc - GndVd + GnnVn = Inn + GnbVb$$

The coefficients and currents of the nodes we determine

$$G_{aa} = \frac{1}{0} + \frac{1}{R_5} + \frac{1}{R_1} = \infty + \frac{1}{5} + \frac{1}{0} = \infty;$$

$$G_{ab} = G_{ba} = \frac{1}{R_5} = \frac{1}{5} = 0.2, \Omega^{-1};$$

$$G_{ac} = G_{ca} = \frac{1}{R_1} = \frac{1}{10} = 0.1, \Omega^{-1};$$

$$G_{ad} = G_{da} = 0, \Omega^{-1};$$

$$G_{an} = G_{na} = \frac{1}{0} = \infty, \Omega^{-1};$$

$$I_{aa} = \frac{E_3}{0} = \infty E_3, A;$$

$$G_{bb} = \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{0} = \infty;$$

$$G_{bc} = G_{cb} = \frac{1}{\infty} = 0, \Omega^{-1};$$

$$G_{bd} = G_{db} = \frac{1}{R_4} = \frac{1}{10} = 0.1, \Omega^{-1};$$

$$G_{bn} = G_{nb} = \frac{1}{\infty} = 0, \Omega^{-1};$$

$$I_{bb} = \frac{-E_2}{0} = -\infty E_2, A;$$

$$G_{cc} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_6} + \frac{1}{\infty} = \frac{1}{10} + \frac{1}{15} + \frac{1}{10} + \frac{1}{\infty} = 0.267, \Omega^{-1};$$

$$G_{cd} = G_{dc} = \frac{1}{R_6} = \frac{1}{10} = 0.1, \Omega^{-1};$$

$$G_{cn} = G_{nc} = 0, \Omega^{-1};$$

$$I_{cc} = J_{S2} = 1, A;$$

$$G_{dd} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_6} + \frac{1}{\infty} = \frac{1}{20} + \frac{1}{10} + \frac{1}{10} + \frac{1}{\infty} = 0.25, \Omega^{-1};$$

$$G_{dn} = G_{nd} = \frac{1}{R_3} + \frac{1}{\infty} = \frac{1}{20} + 0 = 0.05, \Omega^{-1};$$

$$I_{dd} = -J_{S3} = -0.5, A;$$

$$G_{nn} = \frac{1}{R_3} + \frac{1}{0} + \frac{1}{\infty} = \frac{1}{20} + \infty + 0 = \infty, \Omega^{-1};$$

$$I_{nn} = \frac{-E_3}{0} + J_{S3} = -\infty E_3 + 0.5, A.$$

The equations into which we substitute numerical values we can write:

$$\infty Va - 0.1Vc - 0Vd - \infty Vn = \infty E_3 + 0.2Vb;$$
$$-0.2Va - 0Vc - 0.1Vd - 0Vn = -\infty E_2 - \infty Vb;$$
$$-0.1Va + 0.267Vc - 0.1Vd - 0Vn = 1 + 0Vb;$$
$$-0Va - 0.1Vc + 0.25Vd - 0.05Vn = -0.5 + 0.1Vb;$$
$$-\infty Va - 0Vc - 0.05Vd + \infty Vn = -E_3\infty + 0.5A. + 0Vb.$$

We simplify the resulting system of equations by dividing the terms of the first and fifth equations by an infinite value

$$Va – Vn = 20;$$
$$–0.2Va – 0.1Vd = 0;$$
$$–0.1Va + 0.267Vc – 0.1Vd = 1;$$
$$–0.1Vc + 0.25Vd – 0.05Vn = –0.5 – 3;$$
$$–Va + Vn = –20.$$
$$–0.2Vn – 0.1 Vd = 4;$$
$$–0.1Vn + 0.267Vc – 0.1Vd = 3;$$
$$–0.05Vn – 0.1Vc + 0.25Vd = –3.5.$$

After a numerical solution, we get:
$V_n = –23.62V; V_c = –1.92V; V_d = –11.5V;$
$V_a = –3.62V; V_b = –E_2 = –10.0V; V_m = 0V$.
Having indicated the positive directions of the currents in the branches of the circuit, according to Ohm's law we can determine the unknown currents:

$$I_1 = \frac{V_a - V_c}{R_1} = \frac{-3.62 + 1.92}{10} = -0.17A;$$
$$I_{21} = \frac{V_m - V_c}{R_2} = \frac{0 + 1.92}{15} = 0.128A;$$
$$I_2 = I_{21} + J_{S2} = 0.192 + 1 = 1.192A;$$
$$I_{31} = \frac{V_d - V_n}{R_3} = \frac{-11.5 + 23.62}{20} = 0.606A;$$
$$I_3 = I_{31} + J_{S3} = 0.606 + 0.5 = 1.106A;$$
$$I_4 = \frac{V_b - V_d}{R_4} = \frac{-10 + 11.5}{10} = 0.15A;$$
$$I_5 = \frac{V_a - V_b}{R_5} = \frac{-3.62 + 10}{5} = 1.276A;$$
$$I_6 = \frac{V_c - V_d}{R_6} = \frac{-1.92 + 11.5}{10} = 0.958A.$$

In the power balance for energy sources, we take into account the generated (delivered) power with a plus sign (the current and voltage at the source do not coincide in direction), and the power consumption by the source with a minus sign (the current and voltage in this case coincide at the source in the direction). Total power of sources is:

$$\sum_{i=1}^{4} P_i^S = E_2 I_2 + E_3 I_3 - I_{S3} R_3 I_{31} - I_{S2} R_2 I_{21} =$$
$$= 10 \cdot 1.192 + 20 \cdot 1.106 - 0.5 \cdot 20 \cdot 0.606 - 1 \cdot 15 \cdot 0.128 = 26.06W.$$

Total power of loads is:

$$\sum_{j=1}^{6} P_j^L = I_1^2 R_1 + I_{21}^2 R_2 + I_{31}^2 R_3 + I_4^2 R_4 + I_5^2 R_5 + I_6^2 R_6 =$$

$$= 10 \cdot (-0.17)^2 + 15 \cdot 0.128^2 + 20 \cdot 0.606^2 + 10 \cdot 0.15^2 +$$

$$+ 5 \cdot 1.276^2 + 10 \cdot 0.958^2 = 25.423W.$$

Relative computation error is:

$$= \frac{P_S - P_L}{P_S} 100\% = \frac{26.06 + 25.423}{26.06} 100\% = 2.44\%.$$

Engineering calculations are performed with an accuracy of 5 percent.

1.5. Mesh Currents Method Using for Calculating Parameters of DC circuits. Power Sources conversion.

The method is based on the fact that the loop current is independent of the currents flowing in other loops. On this basis, the calculation is made in the following sequence.

1. Number of independent loops in the circuit is deterined. A branch with current sources should enter only one loop. Then the loop current of this loop is known and is equal to the current of the current source.
2. Positive directions of the loop currents in each loop is choosed arbitrarily and mark them on the circuit (I_{11}-loop current of the first loop, I_{22}- second etc.).
3. The system of equations is written in the following form:

$$\begin{aligned}
&R_{11} I_{11} + R_{12} I_{22} + \ldots R_{1k} I_{kk} = E_{11};\\
&R_{21} I_{11} + R_{22} I_{22} + \ldots R_{2k} I_{kk} = E_{22};\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&R_{k1} I_{11} + R_{k2} I_{22} + \ldots R_{kk} I_{kk} = E_{kk}.
\end{aligned}$$

The number of terms on the left side of the equations is equal to the number of loop currents without a current source.

4. If the loop currents of two adjacent loops coincide in direction in adjacent branches, then $R_{12} = R_{21}$, ..., R_{k1} and so on are taken with a plus sign, if not, then a minus sign. Similarly in the direction of energy sources $J_{11}R_{11}$. E_{11}, E_{22}, …E_{kk} are algebraic sums of the first, second, k-th circuits, that is, $E_{11}=\sum Ei$,, where Ei is the EMF of the sources of the first loop, $E_{22}=\sum Ej$, where Ej is EMF of sources of the second loop etc. The sign of Ei, Ej is determined in accordance with positive

direction of the loop current. If the directions of the loop current and EMF coincide, then the EMF is taken in calculations with a "plus" sign, if the directions are opposite, then with a "minus" sign.

5. The system of equations is solved in accordance with unknown parameters (for example, currents I_{11}, I_{22} etc.)

6. The loop currents flowing through the external branches are actually existing ones flowing through these branches and loop currents of internal branches are fictitious quantities introduced for convenience of calculation. The actual currents through the internal branches are found as the algebraic sum of the loop currents that are closed through these branches.

Calculating example # 3: Determine the currents in the branches of the electrical circuit, Fig.1.4, if

J_{S3}=0.5 A; $E_1 = 10V; E_3 = 20V;$
$R_1 = 10\Omega; R_2 = 15\Omega; R_3 = 20\Omega;$
$R_{31} = 5\Omega;\ R_6 = 10\Omega;$
$R_4 = 10\Omega; R_5 = 5\Omega.$

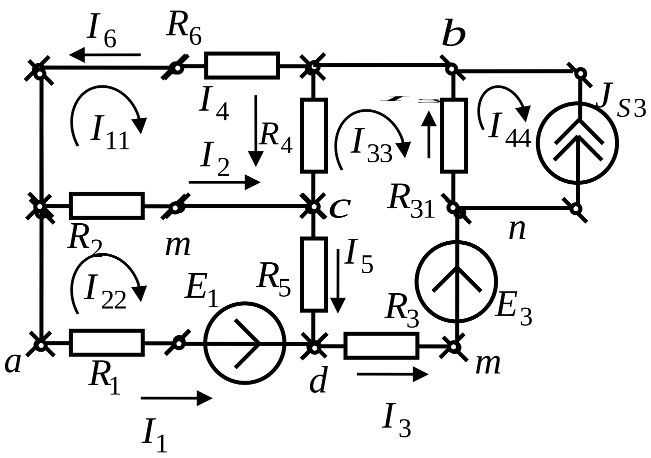


Fig.1.4

Solution Calculating example # 3:
This circuit has 4 independent loops (meshes), each of which is flowed around by single independent mesh current $I_{11}, I_{22}, I_{33}, I_{44}$ of its own. We choose the positive directions of the loop currents and indicate them in the circuit. The loop current I_{44} is known and equals J_{S3} (I_{44}=–J_{S3}). Then one can write the equations for the first, second and third loops in general form:

$$R_{11}I_{11} - R_{12}I_{22} - R_{13}I_{33} - R_{14}I_{44} = E_{11} \qquad (1)$$
$$-R_{21}I_{11} + R_{22}I_{22} - R_{23}I_{33} - R_{24}I_{44} = E_{22} \qquad (2)$$
$$-R_{31}I_{11} - R_{32}I_{22} + R_{33}I_{33} - R_{34}I_{44} = E_{32} \qquad (3)$$

We determine the coefficients for the unknown terms of the equations:

$R_{11}=R_2+R_4+R_6=15+10+10=35\ \Omega$
$R_{12}=R_{21}=R_2=15\ \Omega$
$R_{22}=R_1+R_2+R_5=10+15+5=30\ \Omega$
$R_{13}=R_{31}= R_4=20\ \Omega$
$R_{23}= R_{32}=R_5=5\ \Omega$
$R_{14}= R_{41}=0\ \Omega$
$R_{24}= R_{42}=0\ \Omega$
$R_{34}= R_{43}=R_{31}=5\ \Omega$

$$R_{33}=R_3+R_4+R_5+R_{31}=20+10+5+5=40\ \Omega$$

The voltages drop across adjacent resisters R_{12}, R_{23} ,R_{13} , R_{34} have a "plus" sign because the directions of current in branch with resistors R_{31}, R_2 ,R_4 and R_5 coincide with the selected direction of the loop traversal; the voltage drop across R_{23} has a "minus" sign (-R_4) since the currents I_{22} and I_{44}=J through resistor R_4 are directed in opposite directions relative to the selected traversal direction.

We define loop EMFs:

$$E_{11}=0V$$
$$E_{22}=-E_1=-10\ V$$
$$E33=-E_3=-20\ V$$

After that, one can rewrite equations (1), (2) and (3) in which we replace the coefficients with numerical values:

$$35I_{11}-15I_{22}-20I_{33}-0I_{44}=0$$
$$-15I_{11}+30I_{22}-5I_{33}-0I_{44}=-10$$
$$-20I_{11}-5I_{22}+40I_{33}+5\cdot 0.5=-20$$

Having solved the resulting system of equations, we find the values of the loop (mesh) currents:

$$I_{11}=-0.541\ A$$
$$I_{22}=-0.735\ A$$
$$I_{33}=-0.79\ A$$
$$I_{44}=-0.5\ A$$

Finally, we determine the currents of the branches. First, we indicate the positive directions of the currents in the circuit. Knowing the positive directions, we get:

$$I_1=-I_{22}=0.735\ A$$
$$I_2=-I_{11}+I_{22}=-(-0.541)+(-0.735)=-0.194\ A$$
$$I_3=-I_{33}=-(-0.79\ )=0.79\ A$$
$$I_4=I_{11}-I_{33}=-0.541-(-0.79)=0.249\ A$$
$$I_5=I_{22}-I_{33}=-0.735-(-0.79)=0.055\ A$$
$$I_6=-I_{11}=0.541\ A.$$

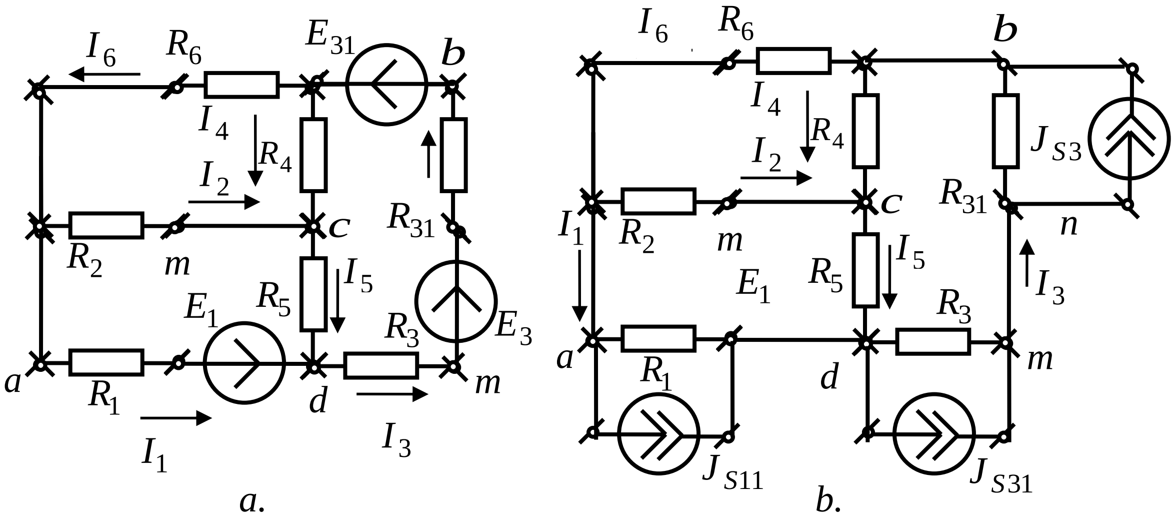


Fig.1.5

If an internal resistance is connected in parallel to an ideal current source, then the ideal current source can be converted into an equivalent ideal voltage source with an internal resistance connected in series. The reverse conversion is also valid. For example, we transform the ideal current source J_{S3} (Fig. 1.4, the initial circuit contains ideal current and EMF sources) into an ideal EMF source (Fig. 1.5.a, the equivalent circuit contains only ideal EMF sources) $E_{31} = R_{31}I_{S1} = 2.5$ V. Or we transform the ideal EMF source (Fig.1.4 the initial circuit has ideal current and EMF sources) into ideal current sources $J_{S11} = J_{S31} =$ (Fig.1.5.b the equivalent circuit contains only ideal current sources). When converting energy sources, it is necessary to take into account that after the conversion of an energy source, its operating mode can be changed i.e. from generating energy, you can go to its consumption and vice versa.

1.6. Superposition Principle Using for DC Circuit Parameters Calculating. Two Node Method

The superposition (overlay) method is based on the condition that a partial current is generated from each energy source in the electrical circuit, regardless of the action of other sources. Real currents in the branches from the action of all energy sources are found as the algebraic sum of partial currents from the action of each source. Based on this condition, the calculation is performed in the following sequence:

1. All energy sources from the circuit are removed, but their internal resistances in the circuit remain, except for only one energy source which is saved in the circuit.
2. Partial currents in the branches from the action of each individual energy source in the circuit are calculated.

3. Positive directions of the currents in each branch are indicated and define these currents as an algebraic sum of partial currents..

Calculating example # 4:

Determine the currents of the branches shown in Fig. 1.6.a of electrical circuit, if J_{S2}=0.5 A; $E_1 = 10V$; $R_1 = 10\Omega; R_2 = 5\Omega; R_3 = 20\Omega;$ $R_4 = 10\Omega; R_5 = 5\Omega$.

Solution Calculating example # 4:

At the first stage, we assume J_{S2}=0 and calculate the partial currents of the branches from the action EMF $E_1 = 10V$. We indicate in the circuit the positive directions of partial currents $I_1', I_2' = I_{21}', I_3'$, that appear from the action of the source $E_1 = 10V$, Fig.1.6.b. The directions of the partial currents are directly determined by the direction of action of the power source E1. These partial currents are determined by one of the previously considered methods, for example, in this case, we can use the two node method as the most rational method. The two node method is a partial case of the nodal potential method.

Potential difference or voltage between circuit nodes

$$U_{bd} = \frac{E_1/(R_1 + R_4)}{1/(R_1 + R_4) + 1/(R_3 + R_5) + 1/R_2} = \frac{0.5}{0.05 + 0.04 + 0.2} = 1.724V.$$

The partial currents of the branches according to Ohm's law are found

$$I_1' = \frac{E_1 - U_{bd}}{R_4 + R_1} = \frac{10 - 1.72}{10 + 10} = 0.414A;$$

$$I_2' = \frac{U_{bd}}{R_2} = \frac{1.72}{5} = 0.344A;$$

$$I_3' = \frac{U_{bd}}{R_3 + R_5} = \frac{1.72}{20 + 5} = 0.0688A;$$

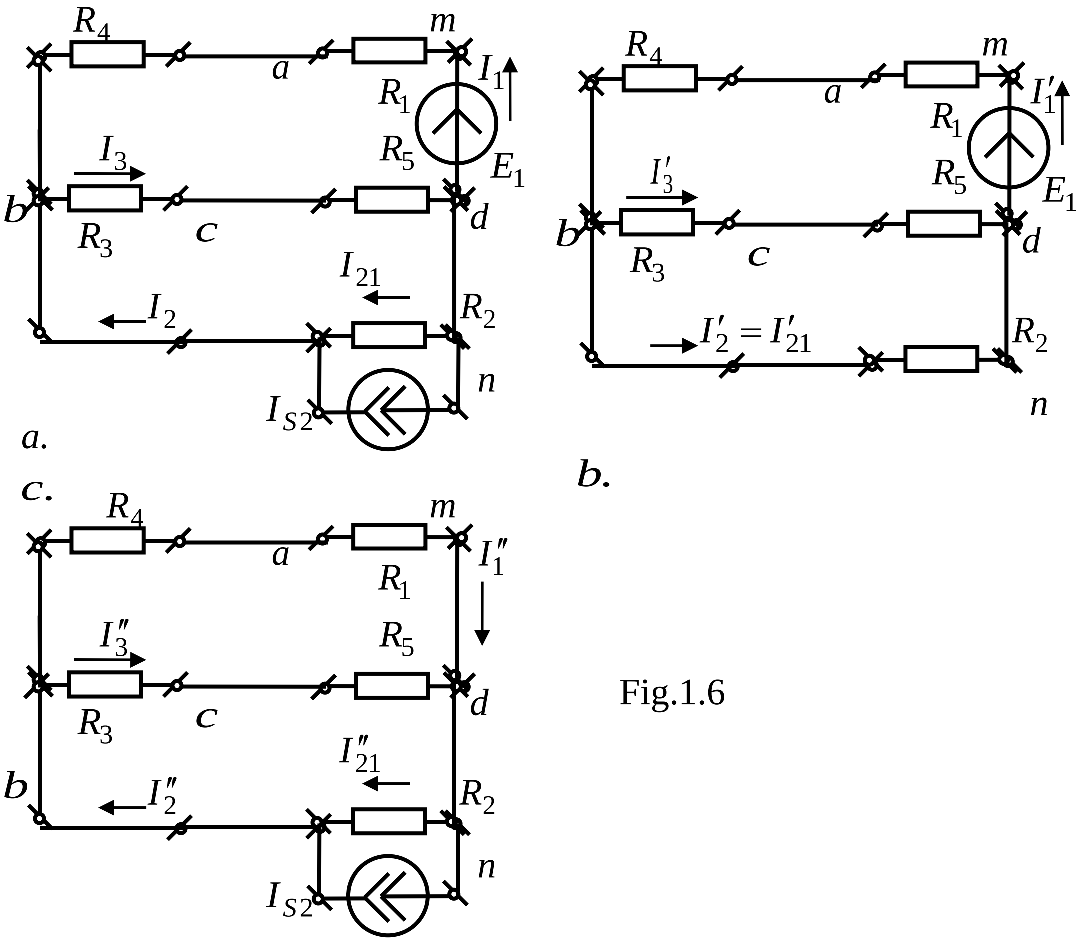


Fig.1.6

At the second stage, we assume $E_1 = 0$ and calculate the currents only from the action of the current source J_{S2}=0.5 A Fig.1.6.c. The calculation is carried out using the method of transforming circuits.
Equivalent resistance of two parallel branches

$$R_e = \frac{(R_1 + R_4)\cdot(R_3 + R_5)}{R_1 + R_4 + R_3 + R_5} = \frac{(10+10)\cdot(20+5)}{10+10+20+5} = 11.11\Omega.$$

According to the rule of dividing currents, we get

$$I''_{21} = -I_{S2}\frac{R_e}{R_e + R_2} = -0.5\frac{11.11}{11.11+5} = -0.345A.$$

The calculation uses the rule of a current divider between two parallel-connected branches: the current in a parallel branch is directly proportional to the product of the input current of the branches by the resistance of the oppositely connected branch and is inversely proportional to the sum of the parallel-connected branches.
Using the first Kirchhoff's law (KCL) we get

$$I''_2 = I_{S2} - I''_{21} = 0.5 - 0.345 = 0.155A;$$

$$I_1'' = I_2'' \frac{R_3 + R_5}{R_3 + R_5 + R_4 + R_1} = 0.1555 \frac{20+5}{20+5+10+10} = 0.0861A;$$
$$I_3'' = I_2'' - I_1'' = 0.155 - 0.0861 = 0.0689A.$$

At the third stage, we find the branch currents as an algebraic sum of the partial branch currents

$$I_1 = I_1' - I_1'' = 0.414 - 0.0861 = 0.3229A;$$
$$I_2 = -I_2' + I_2'' = -0.344 + 0.155 = -0.189A;$$
$$I_{21} = -I_{21}' + I_{21}'' = -0.344 - 0.345 = -0.689A;$$
$$I_3 = I_3' + I_3'' = 0.0688 + 0.0689 = 0.1377A.$$

1.7. Equivalent Generator Method Using for DC Circuit Calculation Parameters

The method is mainly used in cases where it is necessary to determine the current, voltage or power in one branch of the circuit. Then the whole circuit is conventionally divided into two parts: the studied passive two-terminal circuit connected to an active two-terminal network. The branch under study includes a branch of interest in the form of a passive two-pole circuit. The rest of the circuit is referred to an active two-pole circuit and is presented as an active two-pole circuit.

The method is based on the assumption that an active two-pole circuit can be replaced by an equivalent generator. The EMF of the equivalent generator is equal to the voltage at the output of the active two-pole circuit in open circuit mode (open-circuit voltage Uoc), then the internal resistance Roc is equal to the equivalent resistance (input resistance) of the active two-pole circuit in relation to the input terminals.

Using these designations, the current in the investigated branch is determined in accordance with the formula

$$I = \frac{U_{oc}}{R + R_{in}}$$

where R is represents the internal resistance of the branch under study.

The calculation by the method of an equivalent generator is performed in the following sequence.

1. From studied circuit separate a passive two-terminal circuit and an active two-terminal one.
2. The branch under study is separated from the circuit. With one of the known methods for calculating electrical circuits determines the voltage at the terminals to which the branch under study was connected.
3. EMF and currents of electricity sources are taken to be zero, but their internal resistance is taken into account in the calculation and the input resistance is

determined in relation to the terminals where the branch under study is connected.
4. After the open circuit voltage Uoc and the input resistance of the active two-pole circuit Rin have been determined, we determine the current in the branch under study.

Calculating example # 5: Determine the current in the branches *b* - *d* of the electrical circuit Fig. 1.7.a if given
J_{S2}=0.5 A; $E_1 = 10V$; $R_1 = 10\Omega; R_2 = 5\Omega; R_3 = 20\Omega;$ $R_4 = 10\Omega; R_5 = 5\Omega; R_6 = 10\Omega$.

Solution Calculating example # 5: We divide the entire circuit with respect to terminals *b* – *d* into an active two-pole circuit and a passive one (the latter is the studied *branch b* – *d* with known resistance R_6). The studied branch is disconnected from terminals *b* – *d* and get an active two-terminal circuit.

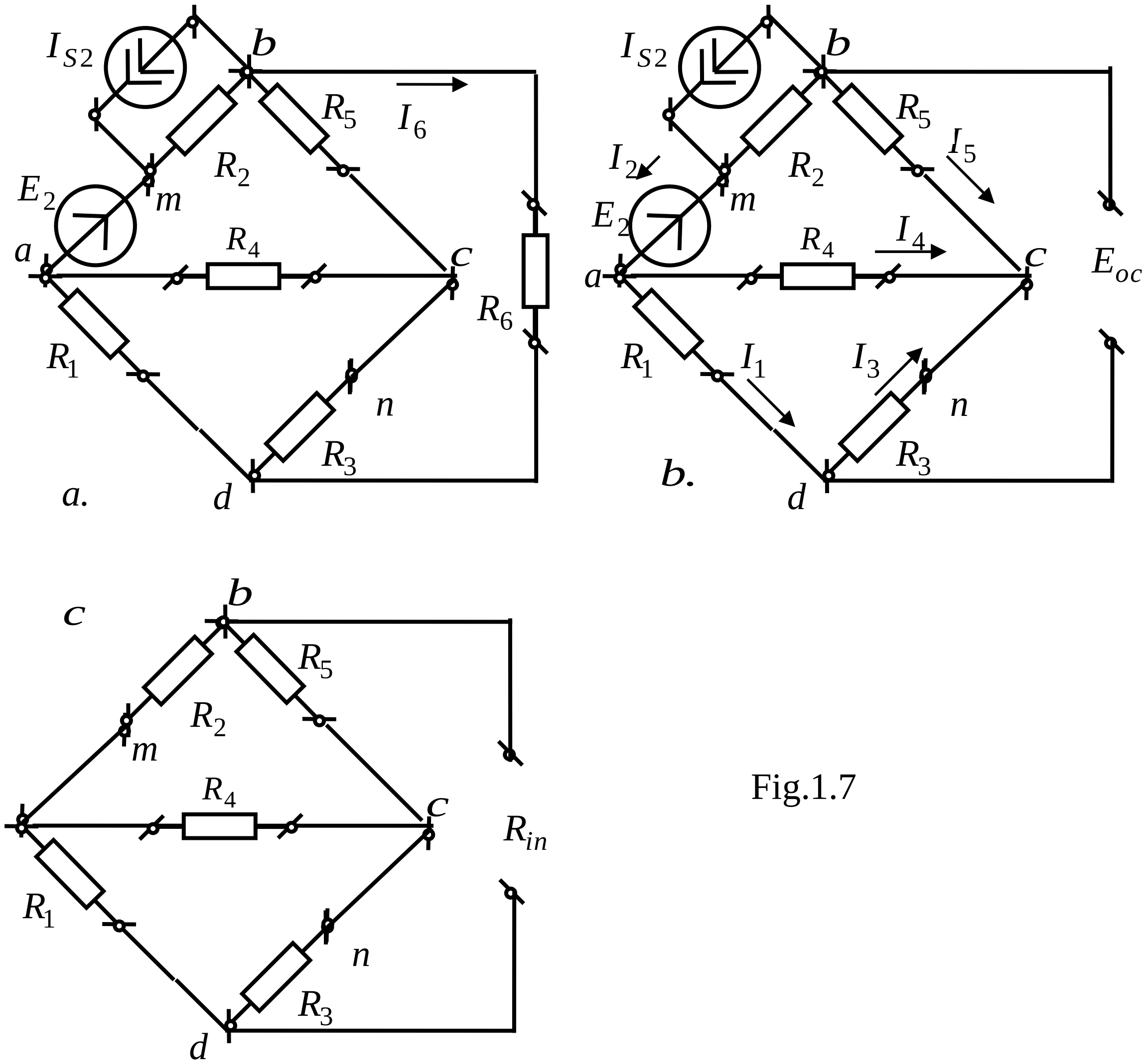


Fig.1.7

At the first stage of the calculation, we find the open circuit EMF Exx Fig. 1.7.b. Using one of the known methods, we determine the potential difference between points *b* – *d*, for example, by the transformation method. The potential difference between points *b* – *d* will be

$$E_{oc} = V_b - V_d = I_5 R_5 - I_3 R_3,$$

where the open circuit current of the two-terminal device Fig. 1.7.b is found as

$$I_2 = -I_5 = \frac{I_{S2}R_2 - E_2}{R_2 + R_5 + \dfrac{R_4 \cdot (R_1 + R_3)}{R_4 + R_1 + R_3}} = \frac{0.5 \cdot 5 - 10}{5 + 5 + \dfrac{10 \cdot (10 + 20)}{10 + 10 + 20}} = -0.428A;$$

$$I_4 = I_2 \frac{R_1 + R_3}{R_1 + R_3 + R_4} = -0.428 \frac{10 + 20}{10 + 20 + 10} = -0.321A;$$

$$I_1 = I_3 = I_2 - I_4 = -0.428 + 0.321 = -0.107A;$$

Then

$$E_{oc} = V_b - V_d = I_5R_5 - I_3R_3 = 0.428 \cdot 5 - (-0.107) \cdot 20 = 4.28V.$$

At the second stage of the calculation, we find the internal resistance Rin of the active two-terminal device Fig. 1.7.c. At first the resistance delta connection is transformed into an equivalent wye connection

We determine the equivalent resistance

$$R_b = \frac{R_2 \cdot R_5}{R_2 + R_5 + R_4} = \frac{5 \cdot 10}{5 + 5 + 10} = 1.25\Omega$$

$$R_c = \frac{R_4 \cdot R_5}{R_2 + R_4 + R_5} = \frac{5 \cdot 10}{5 + 5 + 10} = 1.25\Omega$$

$$R_a = \frac{R_4 \cdot R_2}{R_2 + R_4 + R_5} = \frac{10 \cdot 10}{5 + 5 + 10} = 2.5\Omega$$

Internal resistance of the equivalent generator is

$$R_{in} = R_b + \frac{(R_a + R_1) \cdot (R_c + R_3)}{R_a + R_1 + R_c + R_3} = 1.25 + \frac{(2.5 + 10) \cdot (1.25 + 20)}{2.5 + 10 + 1.25 + 20} = 9.12\Omega.$$

At the third stage of the calculation, using the value of the known open circuit EMF Eoc and the internal resistance of the active two-terminal circuit Rin, we determine I_6 according to Ohm's law

$$I_6 = \frac{E_{oc}}{R_{in}} = \frac{4.28}{9.12} = 0.469A.$$

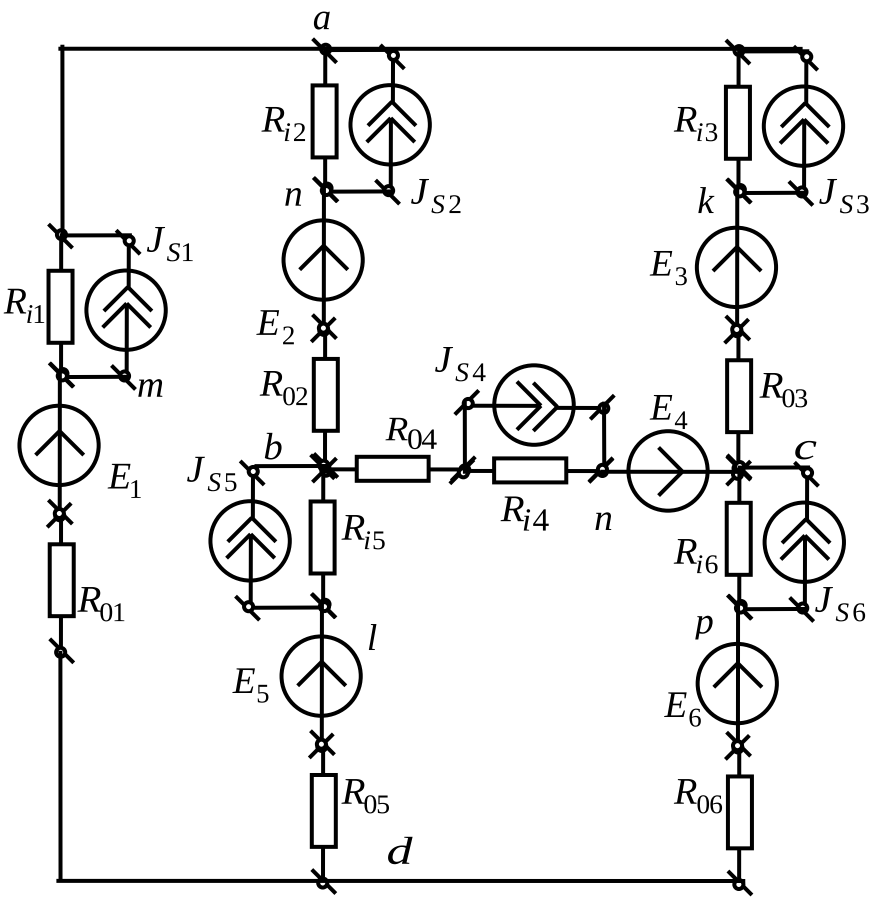
1.8. First Personal Calculating-Graphical Task.
Analysis of Linear DC Circuit

Task. For an electrical circuit that corresponds to the number of the individual variant and is presented in Figure 1.8, the following must be performed

1. According to Kirchhoff's laws, draw up a system of equations for calculating currents in all branches of the circuit.
2. Determine the currents in all branches of the circuit by the method of loop (mesh) currents.
3. Determine the currents in all branches of the circuit by the method of nodal potentials.
4. Summarize the calculation results of currents in a table and compare them with each other.
5. Make up the power balance in the original circuit by calculating the total power of the sources and the total power of the loads (resistances).
6. Determine the current through the resistance in the first branch of the original circuit using the method of the equivalent generator.
7. Build a potential diagram for any closed loop circuit that includes two power sources.

Directions:

1. All internal resistances of the sources are equal to 1 Ohm.
2. The branch to the current source, which in accordance with the initial conditions is equal to zero, in the calculating circuit do not show.
3. The source of EMF, which in accordance with the initial conditions is equal to zero, in the calculating circuit do not show.
4. Indicating the currents in the branches in the diagram, it is necessary to take into account that the current through the resistance, which is parallel to the current source, will differ from the source current.
5. For students whose surname begins with the letters A-E, take the potential of node a as zero potential; with letters Ж-М - potential of node b; with letters Н-Т - potential of node c; with letters У-Я - potential of node d.
6. In clause 6 of the problem, when determining the input resistance of an active two-terminal circuit, the resistance delta connected should be converted into an equivalent wye connected.



a
R_{i2}
J_{S2}
n
R_{i3}
J_{S3}
k
E_3
J_{S1}
R_{i1}
m
E_2
R_{02}
J_{S4}
R_{03}
E_4
R_{04}
b
c
E_1
J_{S5}
R_{i5}
R_{i4}
n
R_{i6}
J_{S6}
R_{01}
p
l
E_5
E_6
R_{05}
R_{06}
d

Fig.1.8

Table 1.1.

Variant	R_{01}	R_{02}	R_{03}	R_{04}	R_{05}	R_{06}	E_1	E_2	E_3	E_4	E_5	E_6	J_{S1}	J_{S2}	J_{S3}	J_{S4}	J_{S5}	J_{S6}
	Ω	Ω	Ω	Ω	Ω	Ω	V	V	V	V	V	V	A	A	A	A	A	A
1	**2**	**3**	**4**	**5**	**6**	**7**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
01	13	5	9	7	10	4	0	0	0	0	10	21	0	0	0	0	0	1
02	13	5	2	8	11	15	0	0	0	0	12	16	0	0	0	0	0	2
03	4	8	6	10	13	10	0	30	0	0	0	9	0	0	0	0	0	1
04	20	80	100	35	150	40	0	100	150	0	0	0	0	0	1	0	0	0
05	10	18	5	10	8	6	0	0	0	0	20	30	0	0	0	0	0	1
06	4	13	9	10	5	6	0	0	0	0	16	8.2	0	0	0	0	0	0.2
07	130	40	60	80	110	45	12	13	0	0	0	0	0	0.3	0	0	0	0
08	6	5	8	14	7	8	20	14	0	0	0	0	1	0	0	0	0	0
09	55	80	100	40	70	120	0	0	0	0	25	10	0	0	0	0	0	0.05
10	110	60	45	150	80	50	0	0	0	25	8	0	0	0	0	0	0.1	-
11	7	12	4	9	15	8	0	0	0	0	20	8	0	0	0	0	0	0.5
12	30	40	22	10	14	50	0	23	9.5	0	0	0	0	0	0.25	0	0	0
13	15	12	10	9	8	7	0	0	0	13	14	0	0	0	0	0.5	0	0
14	12	35	22	6	10	15	0	20	7.6	0	0	0	0	0	0.2	0	0	0
15	4	7	10	12	20	5.5	20	0	0	0	0	10	1	0	0	0	0	0
16	4	11	5	12	7	8	25	4.5	0	0	0	0	0	0.5	0	0	0	0
17	9	20	16	40	30	22	0	0	0	30	10	0	0		0.5	0	0	0
18	5	10	12	7	8	15	0	0	0	0	15	13	0	0	0	0	0	1
19	5	7	10	4	15	20	15	0	20	0	0	0	0	0	1	0	0	0
20	8	10	6	15	21	26	0	0	0	25	0	14	0	0	0	0	0	1
21	19.5	7.5	13,5	10,5	15	6	0	9	45	0	0	0	0	0,8	0	0	0	0

Continuation Table 1.1.

Variant	R_{01}	R_{02}	R_{03}	R_{04}	R_{05}	R_{06}	E_1	E_2	E_3	E_4	E_5	E_6	J_{S1}	J_{S2}	J_{S3}	J_{S4}	J_{S5}	J_{S6}
	Ω	Ω	Ω	Ω	Ω	Ω	V	V	V	V	V	V	A	A	A	A	A	A
1	**2**	**3**	**4**	**5**	**6**	**7**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
43	2	4	3	5	6.5	5	0	0	0	0	11	7.5	0	0	0	0	1	0
44	10	40	50	17.5	75	20	0	0	0	0	34	125	0	0	0	0	0.4	0
45	5	9	2.5	5	4	3	0	0	0	0	8.2	17.5	0	0	0	0	0.2	0
46	2	6.5	4.5	5	2.5	3	0	6.7	5	0	0	0	0	0.2	0	0	0	0
47	65	20	30	40	55	22.5	4.7	12.5	0	0	0	0	0.02	0	0	0	0	0
48	3	2.5	4	7	3.5	4	0	7.5	11	0	0	0	0	1	0	0	0	0
49	27.5	40	50	20	53	0	0	0	0	60	0	6.5	0	0	0	0	0.15	0
50	55	30	22.5	75	40	0	0	0	0	25	8.1	7	0	0	0	0.08	0	0
51	3.5	6	2	4.5	7.5	0	0	0	0	4	0	7	0	0	0	0	0.5	0
52	15	20	11	5	7	25	0	7.5	7.5	0	0	0	0	0.2	0	0	0	0
53	7.5	6	5	4.5	4	3.5	3.5	10	0	0	0	0	0.4	0	0	0	0	0
54	6	17.5	11	3	5	7.5	0	6.5	6	0	0	0	0	0.2	0	0	0	0
55	2	3.5	5	6	10	2.75	0	0	0	0	6.5	10	0	0	0	0	1	0
56	2	5.5	2.5	6	6.5	4	0	0	0	10.5	5	0	0	0	0	1	0	0
57	4.5	10	8	20	15	11	0	0	0	0	1	9	0	0	0	0	0.5	0
58	2.5	5	6	3.5	4	7.5	0	6	12.5	0	0	0	0	0.3	0	0	0	0
59	2.5	3.5	5	2	7.5	10	7	0	15	0	0	0	0.2	0	0	0	0	0
60	4	5	3	7.5	10.5	13	10.5	0	10	0	0	0	0.5	0	0	0	0	0
61	26	10	18	14	20	8	0	0	0	0	20	24	0	0	0	0	0	2
62	26	10	4	16	22	30	0	0	0	0	24	32	0	0	0	0	0	2
63	8	16	12	20	26	20	0	0	0	0	60	24	0	0	0	0	0	0.5

Continuation Table 1.1.

Variant	R_{01}	R_{02}	R_{03}	R_{04}	R_{05}	R_{06}	E_1	E_2	E_3	E_4	E_5	E_6	J_{S1}	J_{S2}	J_{S3}	J_{S4}	J_{S5}	J_{S6}
	Ω	Ω	Ω	Ω	Ω	Ω	V	V	V	V	V	V	A	A	A	A	A	A
1	**2**	**3**	**4**	**5**	**6**	**7**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
22	19.5	7.5	3	12	16.5	22.5	0	12	30	0	0	0	0	0.8	0	0	0	0
23	6	12	9	15	19.5	15	0	0	0	0	21	22	0	0	0	0	2	0
24	30	120	150	52.5	225	60	0	90	375	0	0	0	0	0.5	0	0	0	0
25	15	27	7.5	15	12	9	0	0	0	0	16.5	52.5	0	0	0	0	0.5	0
26	6	19.5	13.5	15	7.5	9	0	16.2	15	0	0	0	0	0.4	0	0	0	0
27	195	60	90	120	165	67.5	0	0	0	10.2	37.5	0	0.04	0	0	0	0	0
28	9	7.5	12	21	10.5	12	0	15	33	0	0	0	0	2	0	0	0	0
29	82.5	120	150	60	105	180	0	0	0	0	25.5	22.5	0	0	0	0	0.1	0
30	165	90	67.5	225	120	75	21	21	0	0	0	0	0.1	0	0	0	0	0
31	10.5	18	6	13.5	22.5	12	0	0	12	0	15	0	0	0	1	0	0	0
32	45	60	33	15	21	75	0	0	0	0	16.5	22.5	0	0	0	0	0.3	0
33	22.5	18	15	13.5	12	10.5	0	0	0	15	30	0	0	0	0	0.2	0	0
34	18	52.5	33	9	15	22.5	0	9	18	0	0	0	0	0.4	0	0	0	0
35	6	10.5	15	18	30	8.25	0	9	30	0	0	0	0	2	0	0	0	0
36	6	16.5	7.5	18	10.5	12	25.5	15	0	0	0	0	2	0	0	0	0	0
37	13.5	30	24	60	45	33	0	15	27	0	0	0	0	1	1	0	0	0
38	7.5	15	18	10.5	12	22.5	0	0	0	0	15	37.5	0	0	0	0	0.5	0
39	7.5	10.5	15	6	22.5	30	0	0	0	15	0	45	0	0	0	1	0	0
40	12	15	9	22.5	31.5	39	25.5	0	30	0	0	0	1	0	0	0	0	0
41	6.5	2.5	4.5	3.5	5	2	0	0	0	0	4	15	0	0	0	0	0.4	0
42	6.5	2.5	1	4	5.5	7.5	0	5	10	0	0	0	0	0.4	0	0	0	0

Continuation Table 1.1.

Variant	R_{01}	R_{02}	R_{03}	R_{04}	R_{05}	R_{06}	E_1	E_2	E_3	E_4	E_5	E_6	J_{S1}	J_{S2}	J_{S3}	J_{S4}	J_{S5}	J_{S6}
	Ω	Ω	Ω	Ω	Ω	Ω	V	V	V	V	V	V	A	A	A	A	A	A
1	**2**	**3**	**4**	**5**	**6**	**7**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
64	40	160	200	70	300	80	0	200	200	0	0	0	0	0	1.5	0	0	0
65	20	36	10	29	16	12	0	40	40	0	0	0	0	0	3	0	0	0
66	8	26	18	20	10	12	0	32	11	0	0	0	0	0	0.5	0	0	0
67	260	80	120	160	220	90	24	34	0	0	40	12	0	0	0	0	0.2	0
68	12	10	16	28	14	16	0	0	0	0	50	22	0	0	0	0	0	2
69	110	160	200	80	140	240	0	0	0	0	40	12	0	0	0	0	0	0.04
70	229	120	90	300	160	100	50	22	0	0	0	0	0	0.05	0	0	0	0
71	14	24	8	18	30	16	0	40	12	0	0	0	0	0	1	0	0	0
72	60	80	44	20	28	100	0	46	8	0	0	0	0	0	0.5	0	0	0
73	30	24	20	18	16	14	0	0	0	26	28	0	0	0	0	0	0.5	0
74	24	70	44	12	20	30	0	0	0	0	40	19.6	0	0	0	0	0	0.1
75	8	124	20	24	40	11	0	0	0	0	40	10	0	0	0	0	0	1.5
76	8	22	10	24	14	16	50	16.6	0	0	0	0	0	0.2	0	0	0	0
77	18	40	32	80	60	44	0	60	28	0	0	0	0	0	0.25	0	0	0
78	10	20	24	14	16	30	0	30	38	0	0	0	0	0	0.5	0	0	0
79	10	14	20	8	30	40	0	0	0	30	0	20	0	0	0	0	0	2
80	16	20	12	30	42	52	0	0	0	50	0	34	0	0	0	0	0	0.5
81	32.5	12.5	22.5	17.5	25	10	0	0	0	0	20	75	0	0	0	0	0.4	0
82	32.5	12.5	5	20	27.5	37.5	0	25	50	0	0	0	0	0.4	0	0	0	0
83	10	20	15	25	32.5	25	0	35	37.5	0	0	0	0	2	0	0	0	0
84	50	200	250	87	375	100	0	150	625	0	0	0	0	0.5	0	0	0	0

Continuation Table 1.1.

Variant	R_{01}	R_{02}	R_{03}	R_{04}	R_{05}	R_{06}	E_1	E_2	E_3	E_4	E_5	E_6	J_{S1}	J_{S2}	J_{S3}	J_{S4}	J_{S5}	J_{S6}
	Ω	Ω	Ω	Ω	Ω	Ω	V	V	V	V	V	V	A	A	A	A	A	A
85	25	45	12.5	25	20	15	0	32	87.5	0	0	0	0	0.4	0	0	0	0
86	10	32.5	22.5	25	12.5	15	0	27	25	0	0	0	0	0.4	0	0	0	0
87	325	100	150	200	275	112	17	62.5	0	0	0	0	0.04	0	0	0	0	0
88	15	12.5	20	35	17.5	20	0	0	0	0	25	55	0	0	0	0	2	0
89	137	200	250	100	175	300	0	0	0	0	34.5	37.5	0	0	0	0	0.14	0
90	275	150	112	375	200	225	0	0	0	24	35	0	0	0	0	0.14	0	0
91	17.5	30	10	22.5	37.5	20	0	26	25	0	0	0	0	0.8	0	0	0	0
92	75	100	55	25	35	125	0	32.5	37.5	0	0	0	0	0.25	0	0	0	0
93	37.5	30	25	22.5	20	17.5	25	50	0	0	0	0	0.2	0	0	0	0	0
94	30	87.5	55	15	25	37.5	0	0	0	0	15	30	0	0	0	0	0.4	0
95	10	17.5	25	30	50	13.5	0	0	0	0	15	50	0	0	0	0	2	0
96	10	27.5	12.5	30	17.5	20	0	0	0	32.5	25	0	0	0	0	3	0	0
97	22.5	50	40	100	75	55	0	35	45	0	0	0	0	0.8	0	0	0	0
98	12.5	25	30	17.5	20	37.5	0	35	62.5	0	0	0	0	0.1	0	0	0	0
99	12.5	17.5	25	10	37.5	50	30	0	75	0	0	0	0.6	0	0	0	0	0
00	20	25	15	37.5	52.5	65	0	0	0	50.5	0	50	0	0	0	0.6	0	0
01	13	5	9	7	10	4	0	0	0	0	10	21	0	0	0	0	0	1
02	13	5	2	8	11	15	0	0	0	0	12	16	0	0	0	0	0	2

1.9. First Personal Calculating-Graphical Task Calculation Example. Analysis of Lineary DC Circuit

Task. For the electrical circuit shown in Figure 1.9, do the following:
1. Draw up a system of equations for calculating currents trough all branches of the circuit according to Kirchhoff's laws.

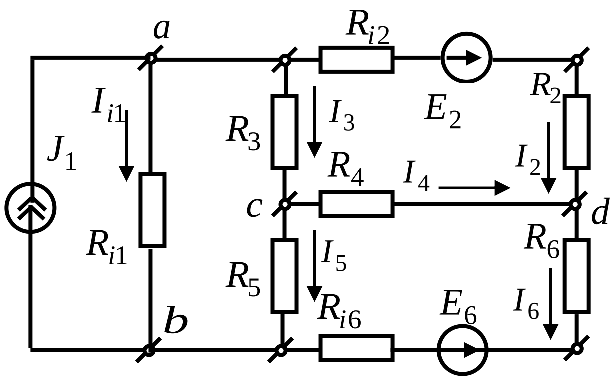


Fig. 1.9

Variant number	Figure number	R_1	R_2	R_3	R_4	R_5	R_6	E_1	E_2	E_6	J_1	I_{S2}	I_{S3}
		Ω						V			A		
1	1.9	13	5	9	7	10	4	-	19	20	1	0	0

$R_{i1} = R_{i1} = R_{i1} = 1\Omega$.

According to KCL: *The current arriving at any junction point in a circuit is equal to the current leaving that junction (Kirchhoff's Current Law):*

for the node *a* : $J_1 - I_{i1} - I_2 - I_3 = 0$;

for the node *c* : $I_3 - I_4 - I_5 = 0$;

for the node *d* : $I_2 + I_4 - I_6 = 0$;

According to KVL: *The sum of the voltage drops around a closed loop is equal to the sum of the source EMFs of that loop (Kirchhoff's Voltage Law).*

for the loop $R_{i1} - R_3 - R_5$: $-R_{i1}I_{i1} + R_3I_3 + R_5I_5 = 0$;

for the loop $R_{i2} - R_2 - R_3$: $I_2(R_{i2} + R_2) - I_3R_3 - I_4R_4 = E_2$;

for the loop $R_{i6} - R_6 - R_5 - R_4$: $-I_6(R_{i6} + R_6) + I_4R_4 - I_5R_5 = -E_6$.

We can describe a system of joint equations according to Kirchhoff's laws, containing six unknown currents:

$$\begin{cases} J_1 - I_{i1} - I_2 - I_3 = 0; \\ I_3 - I_4 - I_5 = 0; \\ I_2 + I_4 - I_6 = 0; \\ -R_{i1} I_{i1} + R_3 I_3 + R_5 I_5 = 0; \\ I_2 (R_{i2} + R_2) - R_3 I_3 - R_4 I_4 = E_2; \\ -I_6 (R_{i6} + R_6) + R_4 I_4 - R_5 I_5 = -E_6 \end{cases}.$$

After substitution of numerical values, we get:

$$\begin{cases} 1 - I_{i1} - I_2 - I_3 = 0; \\ I_3 - I_4 - I_5 = 0; \\ I_2 + I_4 - I_6 = 0; \\ -1 \cdot I_{i1} + 9 \cdot I_3 + 10 \cdot I_5 = 0; \\ I_2 (1+5) - 9 \cdot I_3 - 7 \cdot I_4 = 19; \\ -I_6 (1+4) + 7 \cdot I_4 - 10 \cdot I_5 = -20 \end{cases}.$$

Having solved the system of equations, one can finally obtain:

$$I_{i1} = 1.04A; I_2 = 0.615A; I_3 = -0.654A; I_4 = -1.35A; I_5 = 0.692; I_6 = -0.731A.$$

2. Determination of currents in all branches of the circuit by the method of loop (mesh) currents.

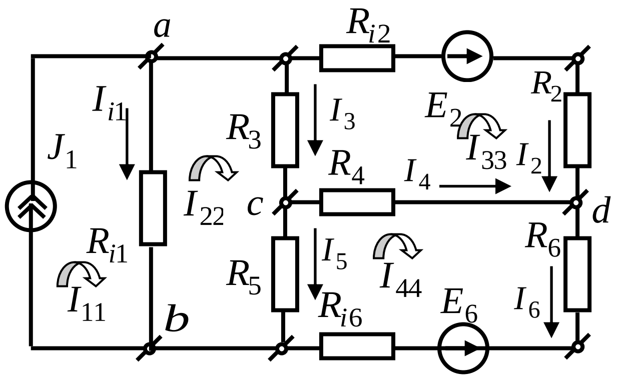


Fig. 1.10

We will demonstrate the application of the basic principles of this method in the circuit of Fig. 1.11, which contains a current source. This circuit contains four independent loops (meshs). Into each mesh we enter the loop curren: I_{11}, I_{22}, I_{33} and I_{44}. All loop currents we are directed "clockwise". For each circuit, we draw up an equation according to the second Kirchhoff's law. With this, it should be noted that the common elements of adjacent branches (branches with resistances R_{i1}, R_3, R_4 and R_5) are flowed around by the difference of currents, respectively $I_{11} - I_{22}, I_{22} - I_{33}, I_{33} - I_{44}$ and $I_{22} - I_{44}$. In this circuit there are no adjacent resistances between 1-3 and 1-4 loops $R_{13} = R_{31} = 0, R_{14} = R_{41} = 0$. The positive direction of the loops traversing is also chosen "clockwise".

For the first loop ($J_1 - R_{i1}$): $I_{11}(R_{i1} + \infty) + (I_{11} - I_{22})R_{i1} = J_1 \infty$.

To get rid of uncertainty (infinity), we divide the left and right sides of the equation by infinity and obtain $I_{11} = J_1$.

For the second loop ($R_{i1} - R_3 - R_5$):

$$-\left(I_{11} - I_{22}\right)R_{i1} + (I_{22} - I_{33})R_3 + \left(I_{22} - I_{44}\right)R_5 = 0.$$

For the third loop ($R_{i2} - R_2 - R_3$):

$$-\left(I_{22} - I_{33}\right)R_3 + (I_{33} - I_{44})R_4 = E_2.$$

For fourth loop ($R_{i6} - R_6 - R_5 - R_4$):

$$-(I_{22} - I_{44})R_5 - (I_{33} - I_{44})R_4 + I_4(R_{i6} + R_6) = -E_6.$$

These equations are transformed into the following form:

$$I_{11} = J_1;$$

$$-I_{11}R_{i1} + I_{22}\left(R_{i1} + R_3 + R_5\right) - I_{33}R_3 - I_{44}R_5 = 0;$$

$$-I_{22}R_3 + I_{33}\left(R_{i2} + R_2 + R_3 + R_4\right) - I_{44}R_4 = E_2;$$

$$-I_{22}R_5 - I_{33}R_4 + I_{44}\left(R_{i6} + R_6 + R_4 + R_5\right) = -E_6.$$

We introduce the following notation:

$R_{i1} + R_3 + R_5 = R_{22}$ – the sum of resistances inside the second loop (own resistance);

$R_{i2} + R_2 + R_3 + R_4 = R_{33}$ – the sum of resistances inside the third loop (own resistance);

$R_{i6} + R_4 + R_5 + R_6 = R_{44}$ – the sum of resistances inside the fourths loop (own resistance);

$R_{12} = R_{21} = -R_{i1}$ - resistance in common branches of two adjacent loops, respectively for the first and second loops (adjacent resistance);

$R_{24} = R_{42} = -R_5$ - resistance in common branches of two adjacent loops, respectively for the second and fourth loops (adjacent resistance);

$R_{34} = R_{43} = -R_4$ - resistance in common branches of two adjacent loops, respectively for the third and fourth loops (adjacent resistance);

$R_{23} = R_{32} = -R_3$ - resistance in common branches of two adjacent loops, respectively for the second and third loops (adjacent resistance).

If the two loops do not have a common branch, then the corresponding resistance is assumed to be zero. Thus, in our case (Fig. 1.10), the first and third loops did not have a common branch. Therefore, resistance $R_{13} = R_{31} = 0, R_{14} = R_{41} = 0$.

EMF E_{11} is a loop EMF, which is equal to the algebraic sum of the EMF in the first loop. This sum includes EMF with a plus sign, whose direction coincides with the direction of traversing the circuit. Similarly, we define: E_{22} is loop EMF of the second loop; E_{33} is loop EMF of the third loop; E_{44} is loop EMF of the fourth loop.

Taking into account the signs, we rewrite the system of equations (1.2) as follows:

$$I_{11}R_{11} + I_{22}R_{12} + I_{33}R_{13} + I_{44}R_{14} = E_{11};$$
$$I_{11}R_{21} + I_{22}R_{22} + I_{33}R_{23} + I_{44}R_{24} = E_{22};$$
$$I_{11}R_{31} + I_{22}R_{32} + I_{33}R_{33} + I_{44}R_{34} = E_{33};$$

$$I_{11}R_{41}+I_{22}R_{42}+I_{33}R_{43}+I_{44}R_{44}=E_{44}.$$

Taking into account in this system that the current $I_{11}=J_1$ and adjacent resistances of the circuits $R_{13}=R_{31}=0, R_{14}=R_{41}=0$, we obtain a simplified system of equations

$$\begin{cases} I_{11}=1; \\ -I_{11}\cdot 1+I_{22}(1+9+10)-I_{33}\cdot 9-I_{44}\cdot 10=0; \\ -I_{22}\cdot 9+I_{33}(1+5+9+7)-I_{44}\cdot 7=19; \\ -I_{22}\cdot 10-I_{33}\cdot 7+I_{44}(1+4+7+10)=-20. \end{cases}$$

Unknown loop currents are determined from the solution of the last equations

$I_{11}=1A; I_{22}=-0.0385A; I_{33}=0.615A; I_{44}=-0.731A$

and then the currents in the branches are calculated through the known loop currents (Fig.1.9):

$I_{i1}=J_1-I_{22}=1+0.0385=1.0385A;$

$I_2=I_{33}=0.615A;$

$I_3=I_{22}-I_{33}=-0.0385-0.615=-0.6535A;$

$I_4=I_{44}-I_{33}=-0.731-0.615=-1.346A;$

$I_5=I_{22}-I_{44}=-0.0385+0.731=0.6925A;$

$I_6=I_{44}=-0.731A.$

3. Determination of circuit branch currents by the method of nodal potentials.

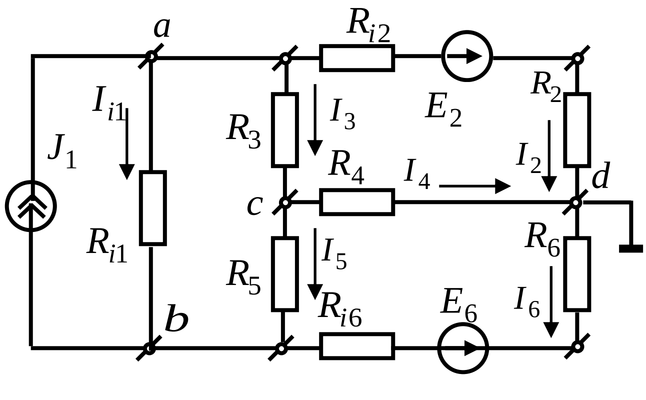


Fig. 1.11

The *d* node of the circuit as a reference is selected. Circuit *d* node we are electrically grounded, and for the remaining *a*, *b*, *c* nodes of the circuit, we make up a system of three equations.

For *a* node, the calculating equation is written in the following form

$$V_a(G_{i1}+G_{i2}+G_3)-V_bG_{i1}-V_cG_3=J_1-E_2G_2$$

$$\text{or} \quad V_a G_{aa} - V_b G_{ab} - V_c G_{ac} = \sum_a EG,$$

where the following notation is introduced:

$J_1 - E_2 G_2 = I_{aa} = \sum_a EG -$ nodal current of *a* node, which is defined as the algebraic sum of the products of EMF and its branch conductance and a current source is added;

$G_{i1} + G_{i2} + G_3 = G_{aa} -$ the sum of the conductance of the branches that are connected to *a* node;

$G_{i1} = G_{ab}$- the sum of the conductance of the branches that are connected between *a* and *b* nodes;

$G_3 = G_{ac}$- the sum of the conductance of the branches that are connected between *a* and *c* nodes.

In a similar way, we write down the equations for *b* and *c* nodes:

$$-V_{ba} G_{aa} + V_b G_{bb} - V_c G_{bc} = \sum_b EG;$$
$$-V_{ca} G_{aa} - V_b G_{cb} - V_c G_{cc} = \sum_c EG.$$

By jointly solving the equations, we determine the values of the potentials of the nodes V_a, V_b, V_c (the potential V_d = *0*), after which one can calculate the currents through the branches.

It should be noted that when we find the algebraic sum of the nodal currents, then if the EMF is directed to the node, then the positive sign is taken and if it is directed from the node, then the negative sign is taken. If the branch does not have an EMF source, the corresponding term will be equal to zero.

Substituting numerical values in equations

$$V_a\left(\frac{1}{1}+\frac{1}{1+5}+\frac{1}{9}\right) - V_b\,\frac{1}{1} - V_c\,\frac{1}{9} = 1 - \frac{19}{1+5};$$

$$-V_a\,\frac{1}{1} + V_b\left(\frac{1}{1}+\frac{1}{10}+\frac{1}{1+4}\right) - V_c\,\frac{1}{10} = -1 - \frac{20}{1+4};$$

$$-V_a\,\frac{1}{9} - V_b\,\frac{1}{10} + V_c\left(\frac{1}{9}+\frac{1}{7}+\frac{1}{10}\right) = 0.$$

From the numerical solution of the last system of equations, we find the potentials of the *a*, *b*, *c* nodes

$$V_a = -15.3V; V_b = -16.3V; V_c = -9.42V; V_d = 0V.$$

After calculating the nodal potentials, one can find the currents in the branches of the circuit according to Ohm's law:

$$I_{i1} = (V_a - V_b)G_{i1} = \frac{-15.3 + 16.3}{1} = 1.0A;$$
$$I_2 = (V_a + E_2)G_2 = \frac{-15.3 + 19}{1+5} = 0.616A;$$
$$I_3 = (V_a - V_c)G_3 = \frac{-15.3 + 9.42}{9} = -0.6533A;$$
$$I_4 = V_c G_4 = \frac{-9.42}{7} = -1.345A;$$
$$I_5 = (V_c - V_b)G_5 = \frac{-9.42 + 16.3}{10} = 0.688A;$$
$$I_6 = (-V_b - E_6)G_6 = \frac{16.3 - 20}{1+4} = -0.74A.$$

4. The calculating the current results are summarized in a table and compared one another.

Calculation method	I_{i1}, A	I_2, A	I_3, A	I_4, A	I_5, A	I_6, A
Kirchhoff's laws	1.04	0.615	-0.654	-1.35	0.692	-0.731
mesh currents	1.0385	0.615	-0.6535	-1.346	0.6925	-0.371
Nodal potentials	1.0	0.616	-0.6533	-1.345	0.688	-0.74

5. Drawing up the powers balance in the initial circuit.
Generated (delivered) power

$$P_S = J_1 \cdot I_{i1} \cdot R_{i1} + E_2 I_2 + E_6 I_6 = 1 \cdot 1.04 \cdot 1 + 19 \cdot 0.615 -$$
$$-20 \cdot (-0.731) = 27.345Wt.$$

Power consumption

$$P_L = I_{i1}^2 R_{i1} + (R_{i2} + R_2) \cdot I_2^2 + R_3 \cdot I_3^2 + R_4 \cdot I_4^2 + R_5 \cdot I_5^2 + (R_{i6} + R_6) \cdot I_6^2 =$$
$$= 1 \cdot 1{,}04^2 + (1+5) \cdot 0{,}615^2 + 9 \cdot 0.654^2 + 7 \cdot 1.35^2 + 10 \cdot 0.692^2 +$$
$$+ (1+4) \cdot 0.731^2 = 27.418Wt.$$

Relative computation error is:

$$\frac{P_L - P_S}{P_L} \cdot 100\% = \frac{27.418 - 27.345}{27.418} \cdot 100\% = 0.266\%.$$

Engineering calculations are performed with an accuracy of 5 percent.

6. Determination of the current I_{i1} through the resistance R_{i1} in the first branch of the original circuit by the equivalent generator method.

Calculation of the internal resistance of the equivalent generator. At the first stage of the calculation, we remove all power sources from the circuit (Figure 1.12, b), but save their internal resistances in the circuit.

In order to simplify the calculation of the electrical circuit, we transform the resistance delta connection $R_{i2}+R_2, R_3, R_4$ (Figure 1.12, b) into a resistance wye connection R_a, R_b, R_c (Figure 1.12, c) or perform the reverse transformation of the wye into a delta

$$R_a = \frac{(R_{i2}+R_2)\cdot R_3}{R_{i2}+R_2+R_3+R_4} = \frac{(1+5)\cdot 9}{1+5+9+7} = 2.45\Omega;$$

$$R_b = \frac{(R_{i2}+R_2)\cdot R_4}{R_{i2}+R_2+R_3+R_4} = \frac{(1+5)\cdot 7}{1+5+9+7} = 1.91\Omega;$$

$$R_c = \frac{R_3\cdot R_4}{R_{i2}+R_2+R_3+R_4} = \frac{7\cdot 9}{1+5+9+7} = 2.86\Omega.;$$

$$R_{in} = \frac{(R_b+R_6+R_{i6})\cdot(R_c+R_5)}{R_b+R_6+R_{i6}+R_c+R_5} + R_a = \frac{(1.91+4+1)\cdot(2.86+10)}{1.91+4+1+2.86+10} + 2.45 = 6.94\Omega.$$

At the second stage of the calculation, we determine the open circuit EMF of the equivalent generator (Figure 1.12, d).
The open circuit current of the equivalent generator is found by the method of loop currents.

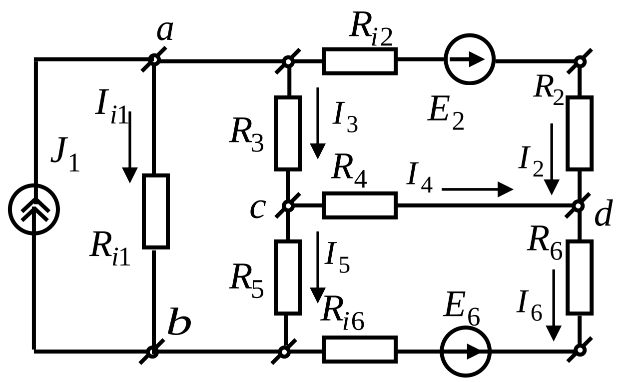


Fig.1.12.a

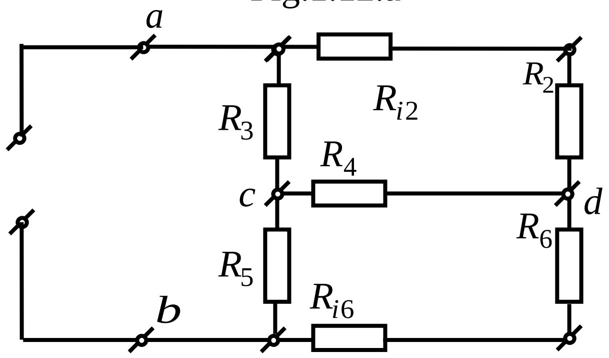


Fig.1.12.b

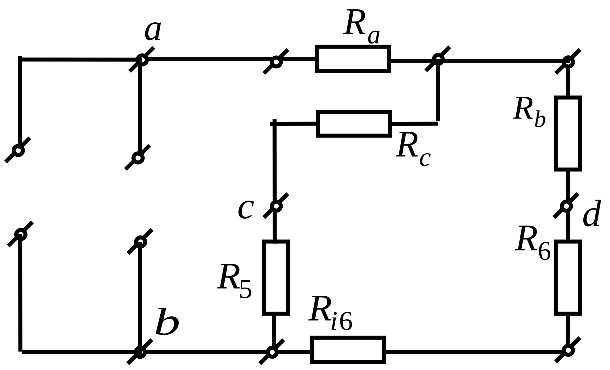


Fig.1.12.c

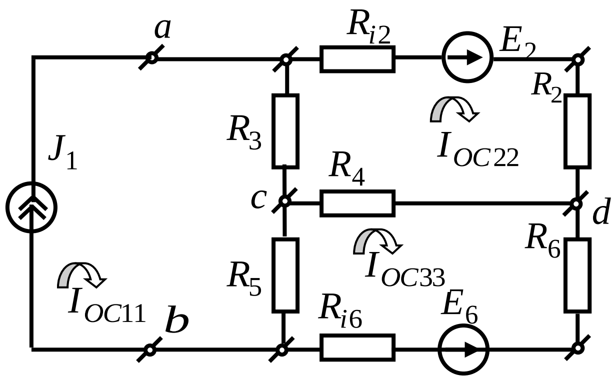


Fig.1.12.d

The open circuit current of the equivalent generator

$I_{OC11} = J_1$;

$-I_{OC11}R_3 + I_{OC22}\left(R_{i2} + R_2 + R_3 + R_4\right) - I_{OC33}R_4 = E_2$;

$-I_{OC11}R_5 - I_{OC22}R_4 + I_{4OC33}\left(R_{i6} + R_6 + R_4 + R_5\right) = -E_6$.

$$\begin{cases} I_{OC11} = 1; \\ -I_{OC11} \cdot 9 + I_{OC22}(1+5+9+7) - I_{OC33} \cdot 7 = 19; \\ -I_{OC11} \cdot 10 - I_{OC22} \cdot 7 + I_{OC33}(1+4+7+10) = -20. \end{cases}$$

$$I_{OC22} = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 28 & -7 \\ -10 & 22 \end{vmatrix}}{\begin{vmatrix} 22 & -7 \\ -7 & 22 \end{vmatrix}} = 1.255A; I_{OC33} = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 22 & 28 \\ -7 & -10 \end{vmatrix}}{\begin{vmatrix} 22 & -7 \\ -7 & 22 \end{vmatrix}} = -0.055A;$$

The open circuit EMF of the equivalent generator

$$E_{OCab} = V_a - V_b = (I_{OC11} - I_{OC22}) \cdot R_3 + (I_{OC11} - I_{OC33}) \cdot R_5 =$$
$$= (1 - 1.255) \cdot 9 + (1 - 0.055) \cdot 10 = 8.255V;$$

The sought current in the first branch

$$I_{i1} = \frac{E_{OCab}}{R_{in} + R_{i1}} = \frac{8.255}{6.94 + 1} = 1.039A.$$

7. To drawing a potential diagram for a d-c-a-d closed loop, which includes two EMF sources E_2 and E_6.

The potentials of the circuit nodes are calculated

$$V_d = 0; V_{R6} = V_d - I_6 \cdot R_6 = V_d + 0.74 \cdot 4 = V_d + 2.96V;$$
$$V_{E6} = V_{R6} - E_6 = V_{R6} - 20V; V_b = V_{E6} - I_6 \cdot R_{i6} = V_d + 0.74 \cdot 1 = V_d + 0.74V;$$
$$V_c = V_b + I_5 \cdot R_5 = V_b + 0.688 \cdot 10 = V_b + 6.88V;$$
$$V_a = V_c + I_3 \cdot R_3 = V_c + I_3 \cdot R_3 = V_c - 0.6533 \cdot 9 = V_c - 5.879V;$$
$$V_{Ri2} = V_a - I_2 \cdot R_{i2} = V_2 - 0.616 \cdot 1 = V_a - 0.616V;$$
$$V_{E2} = V_{Ri2} + E_2 = V_{Ri2} + 19V;$$
$$V_d = V_{E2} - I_2 \cdot R_2 = V_{E2} - 0.616 \cdot 5 = V_{E2} - 3.08V.$$

Based on the found potentials of the nodes, the values of the EMF and resistances of the branches, we build a potential diagram in Fig. 1.13.

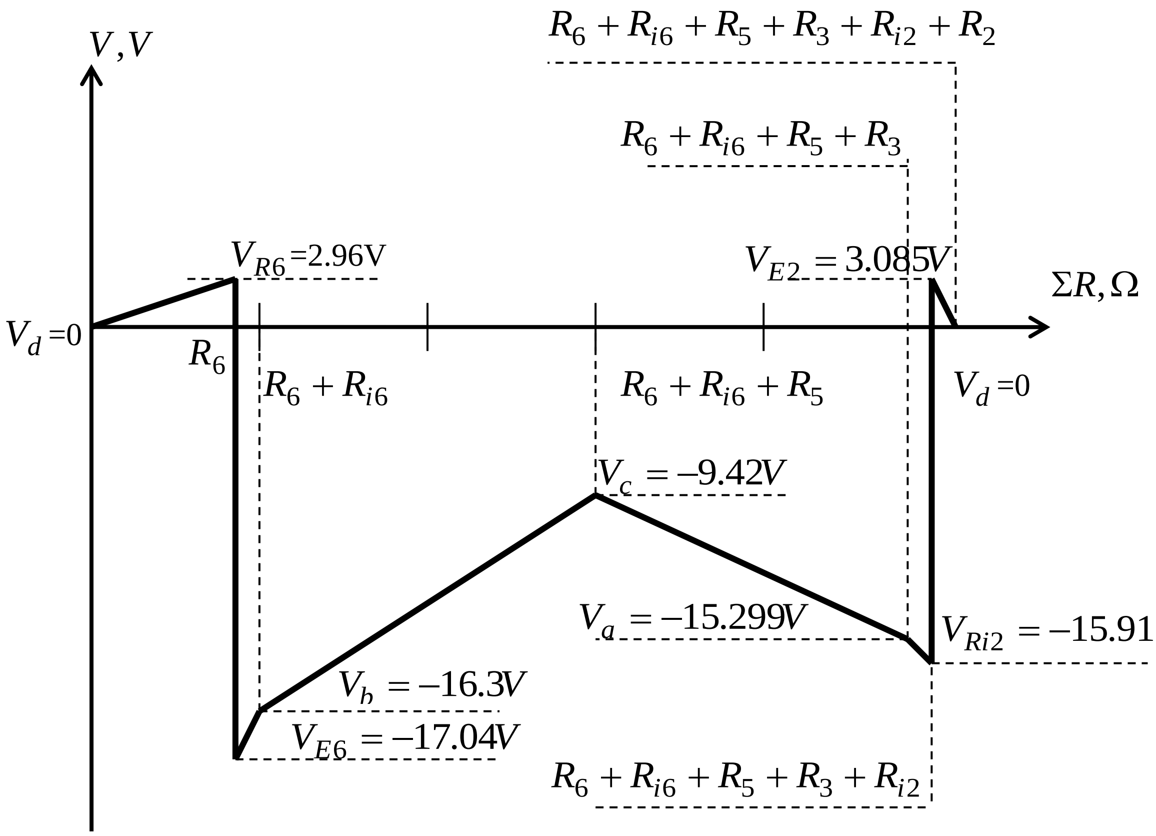
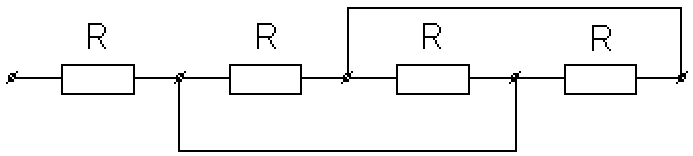


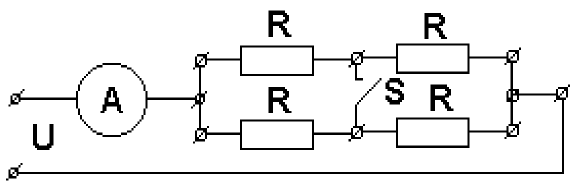
Fig.1.13

1.10. Tasks for self-control of knowledge for DC circuits

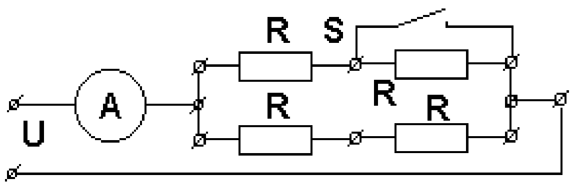
1. Calculate the equivalent resistance of circuit, if R=30 Ω.



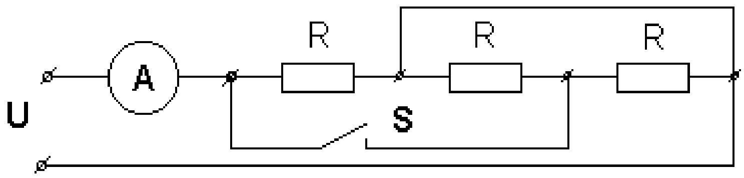
2. How the ammeter readings change if the S key is closed? The input voltage U is constant.



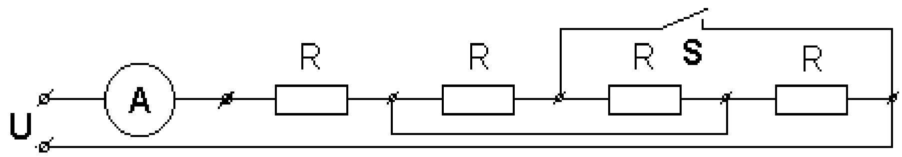
3. What will the ammeter A show if the S key is closed? Before the switch S was closed, the ammeter A reads 9 A, and the input voltage was constant.



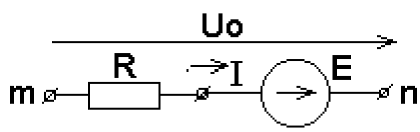
4. Before the key S is closed, the ammeter reading is 6 A. What will the ammeter reading after the S key is closed? Input voltage U is constant.



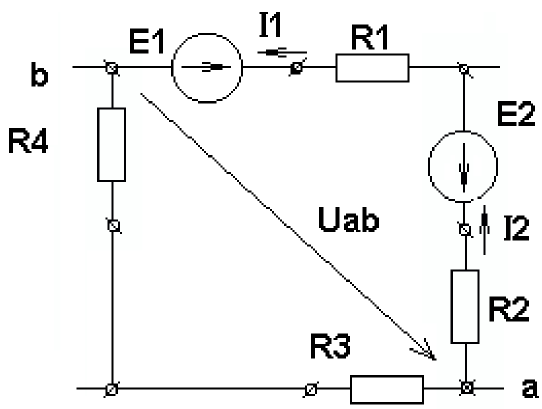
5. Before the key S is closed, the ammeter reading is 2A. What will the ammeter reading after closing the S key? The input voltage U is constant.



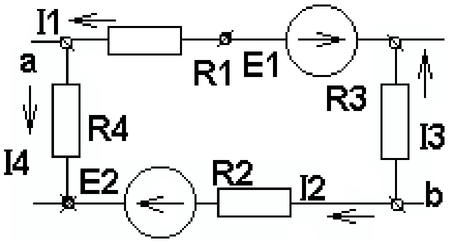
6. A DC branch *mn* is given. Determine the current *I* in the branch *mn* through general through the parameters of the branch branch *E*, U_o, *R*.



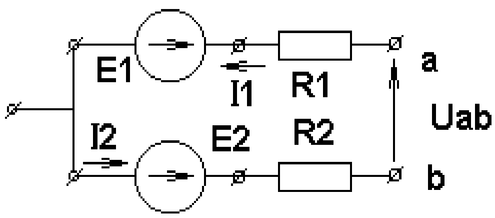
7. A circuit of a direct current circuit is given. Express the voltage Uab in terms of values E1, R1, I1, E2, R2, I2.



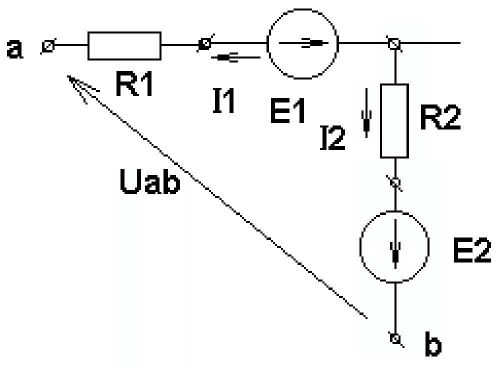
8. A circuit of a direct current circuit is given. Draw up Kirchhoff's Second Law.



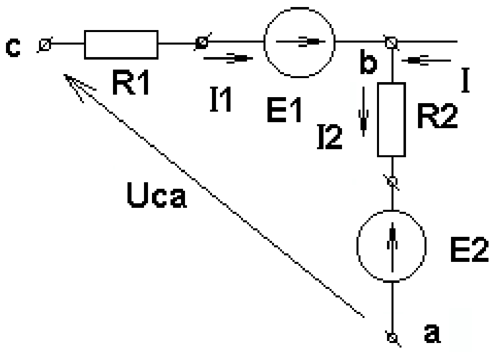
9. The figure shows part of a DC circuit. Given: I1 = 3 A, I2 = 2.4 A, E1 = 70 V, E2 = 20 V, R1 = 8 ohms, R2 = 5 ohms. Find voltage Uab.



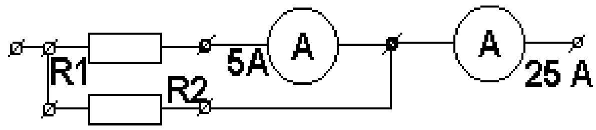
10. Write the voltage Uab through the parameters I1, I2, R1, R2, E1, E2.



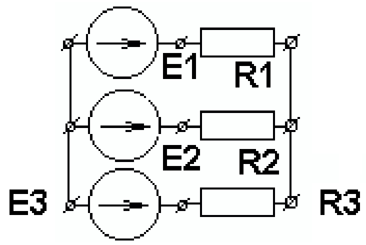
11. The figure shows a section of a complex DC circuit. Make equations according to Kirchhoff's laws and find the branch currents if E1=100 V, E2=130 V, I=8 A, R1=3 Ω, R2=5Ω, Uca=70V.



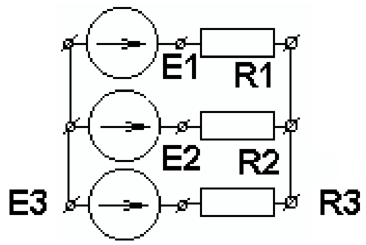
12. Find the resistance R2 for direct current, if R1 = 3 ohms, the ammeter readings are shown in the figure.



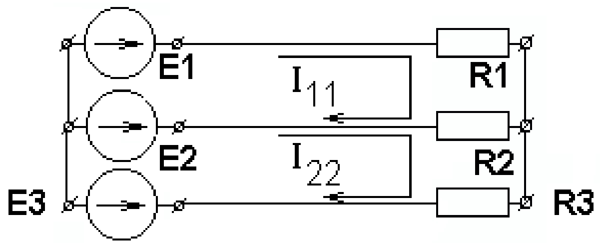
13. Determine which of the three sources of EMF generate energy and which of them consumed this energy, if R1=6 Ω, R2=8 Ω, R3=3 Ω, E1=10 V, E2=E3=30 V.



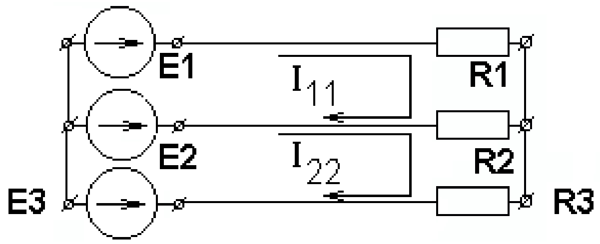
14. Determine the operating modes of the three EMF sources, if R1=6 Ω, R2=8Ω, R3=3 Ω, E1=30 V, E2=10 V, E3=5 V.



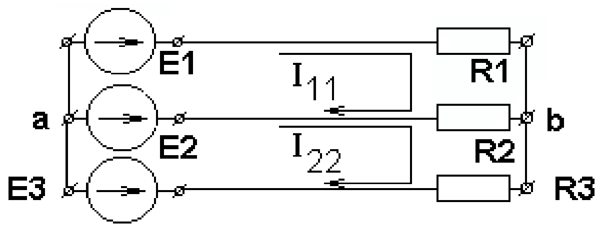
15. Using the loop current method, find the current trough the first source if R1=6 Ohm, R2=8 Ohm, R3=3 Ohm, E1=10 V, E2=20 V, E3=30 V.



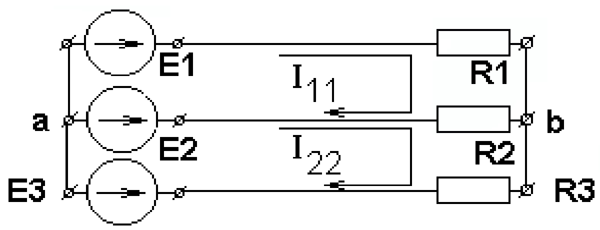
16. Using the loop current method, find the current trough the second source if R1=6 Ohm, R2=8 Ohm, R3=3 Ohm, E1=10 V, E2=20 V, E3=30 V.



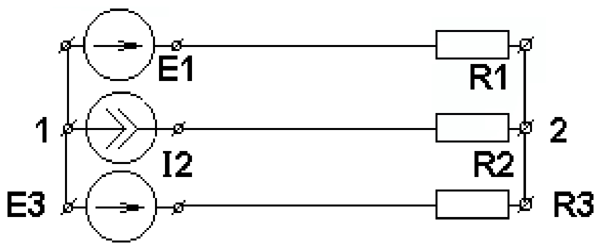
17. Draw up in general form the equation of nodal potentials for node *a* in the case when point *b* of the circuit is grounded. Find in general the current of the first energy source.



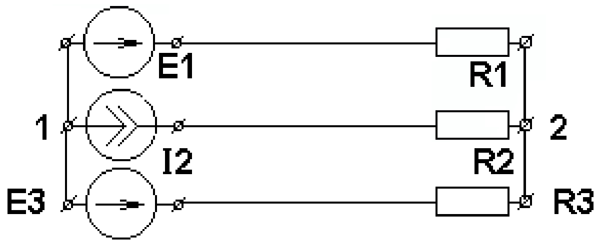
18. Draw up in general form the equation of nodal potentials for node *b* in the case when point *a* of the circuit is grounded. Find in general the current of the second energy source.



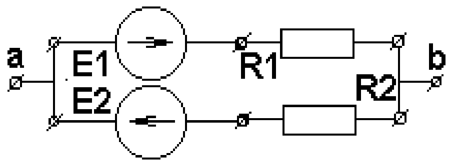
19. Draw up in general form the equation of nodal potentials for node 1 in the case when point *2* of the circuit is grounded. Find in general the current of the third energy source.



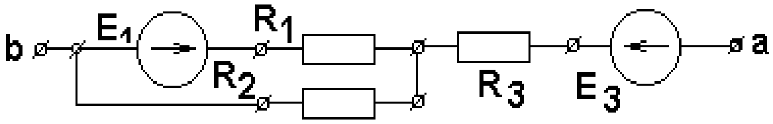
20. Draw up in general form the equation of nodal potentials for node *2* in the case when point *1* of the circuit is grounded. Find in general the current of the first energy source.



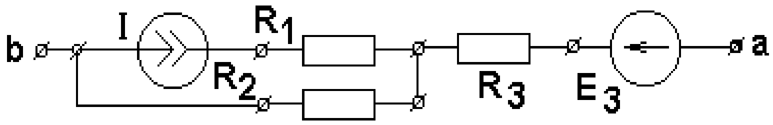
21. Find in the equivalent circuit of a two-terminal device the EMF of the equivalent generator for the original circuit shown in the figure, if E1=50 V, E2=70 V, R1=15 Ω, R2=9 Ω.



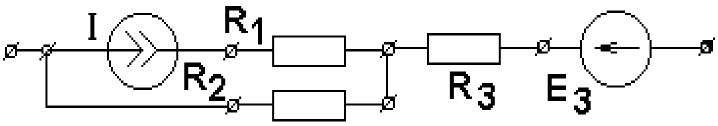
22. Given E1=54 V, E3=36 V, R1=9 Ω, R2=18 Ω, R3=5 Ω . Find in the equivalent circuit of a two-terminal device the EMF of the equivalent generator for the original circuit shown in the figure.



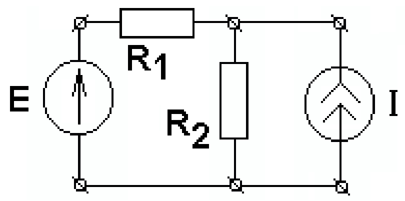
23.Given I=2 A, R1=9 Ω, R2=18 Ω, R3=5Ω, E3=12 V. Find the circuit internal resistance of the two-pole of the equivalent generator for the original circuit shown in the figure.



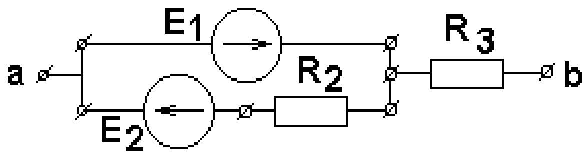
24.Given I=2 A, R1=9 Ω, R2=12 Ω, R3=5 Ω, E3=12 V. Find the EMF of the two-pole of the equivalent generator for the original circuit shown in the figure.



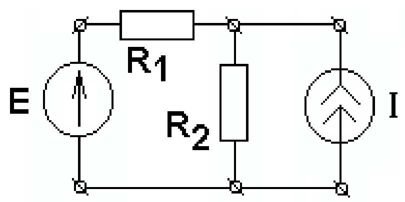
25. Using the loop current method, find the branch currents if E=10 V, I=1 A, R1=R2=5 Ω.



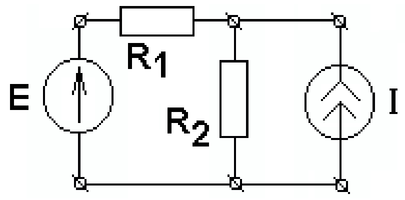
26. Find in the equivalent circuit of a two-terminal device the EMF of the equivalent generator for the original circuit shown in the figure, if E1=50 V, E2=70 V, R2=10 Ω, R3=20Ω.



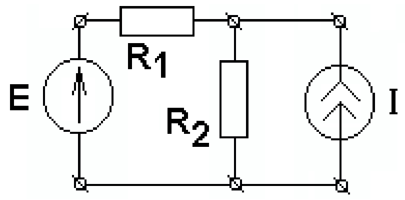
27. Using the method of nodal potentials, find the branch currents if: E=10 V, I=1 A, R1=R2=5 Ω.



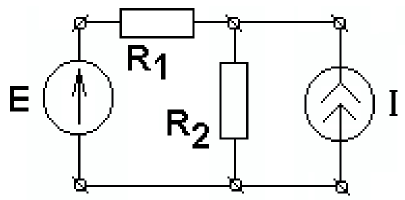
28. Using the superposition method, find the branch currents if E=10 V, I=1 A, R1=R2=5 Ω.



29. Using the equivalent generator method, find the current through the resistor R2, if E=10 V, I=1 A, R1=R2=5 Ω.



30. Using Kirchhoff’s laws, find branch currents: E=10 V, R1=R2=5 Ω, I=1 A.



2. The parameters calculation of linear single-phase AC circuit

2.1. Methods of analysis of the AC circuits

We know that Ohm's and Kirchhoff's laws are applicable to AC circuits. In this section, we want to see how nodal analysis, mesh analysis, superposition and equivalent generator are applied in analyzing AC circuits. Since these techniques were already introduced for DC circuits, our major effort here will be to illustrate with examples.

Frequency-domain analysis of an AC circuit via phasors is much easier than an alysis of the circuit in the time domain.

Analyzing AC circuits usually requires three steps.

Steps to Analyze AC Circuits:

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
3. Transform the resulting phasor to the time domain.

Step 1 is not necessary if the problem is specified in the frequency domain. In step 2, the analysis is performed in the same manner as DC circuit analysis except that complex numbers are involved.

To define currents in the branches of electrical circuit Fig.2.1, if

$e_1(\omega t+\psi_1)=10\sin(\omega t+20^o)V; e_3=20\cos(\omega t+20^o)=20\sin(\omega t+90^o+20^o)V;$
$f=50Hz;\ R_1=10\Omega; R_2=15\Omega; R_3=20\Omega;\ L_1=10mH;\ L_2=20mH;\ C_2=500\mu F;$
$C_3=400\mu F..$

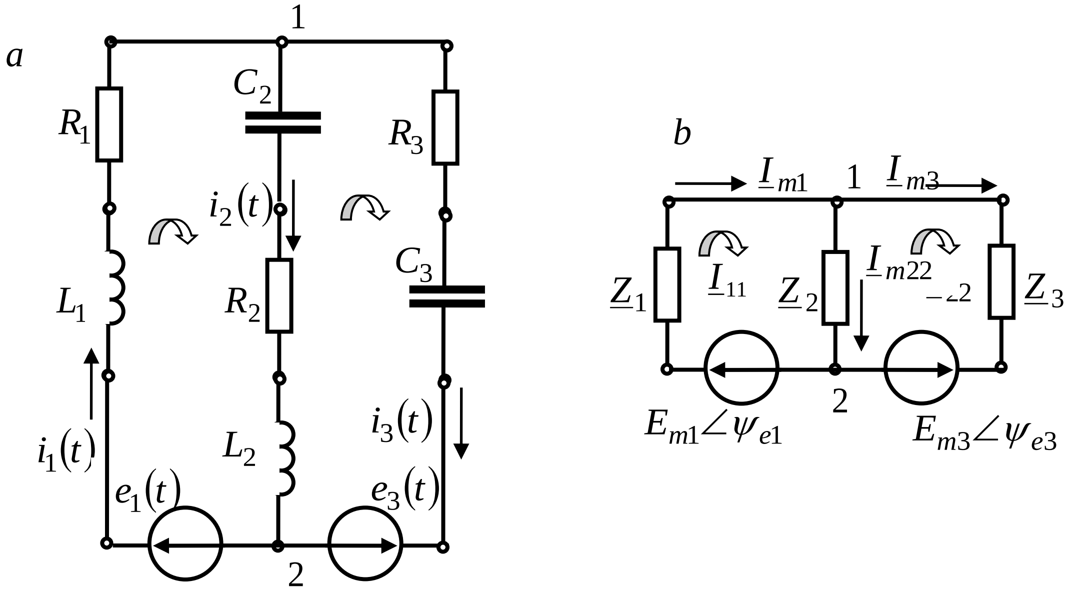


Fig.2.1

1. Time-domain representation. Under KCL and KVL for the scheme 2.1.a we have three independent equations

$$\left.\begin{aligned}
&i_1(t)-i_2(t)-i_3(t)=0;\\
&L_1\frac{di_1}{dt}+i_1R_1+\frac{1}{C_2}\int i_2dt+i_2R_2+L_2\frac{di_2}{dt}=e_1(t);\\
&-L_2\frac{di_2}{dt}-i_2R_2-\frac{1}{C_2}\int i_2dt+i_3R_3+\frac{1}{C_3}\int i_3dt=-e_3(t),
\end{aligned}\right\}$$

where $e_3(t)=E_{m3}\sin(\omega t+\psi_{e3})$; $e_1(t)=E_{m1}\sin(\omega t+\psi_{e1})$; $\omega=2\pi f=314 rad/s$; $i_1(t)=I_{m1}\sin(\omega t+\psi_{i1})$; $i_2(t)=I_{m2}\sin(\omega t+\psi_{i2})$; $i_3(t)=I_{m3}\sin(\omega t+\psi_{i3})$.

We obtained integrodifferential equations. We can find solutions using this system equations, but that is hard way. Direct concluding simultaneous equations bring to bulky conclusions.

2. For the reduction calculus we use phasor-domain representation. Equivalent circuit in phasor-domain representation is affording Fig.2.1.b, where we employ following phasor-domain:

the voltage sources are $\underline{E}_{m1}=E_{m1}e^{j\psi_{e1}}=10e^{j20^o}V=9.39+j3.42V$;

$\underline{E}_{m3}=E_{m3}e^{j\psi_{e3}}=20e^{j110^o}V=-6.84+j18.79V$;

the branch impedances are

$$\underline{Z}_1=Z_1e^{j\psi_{Z1}}=R_1+j\omega L_1=10+j314\cdot10\cdot10^{-3}=$$

$$=\sqrt{10+3.14}e^{jarctg(3.13/10)}=10.48e^{j17.4^o}\Omega=10+3.14j\Omega;$$

$$\underline{Z}_2=Z_2e^{j\psi_{Z2}}=R_2+j\omega L_2-j\frac{1}{\omega C_2}=15+j314\cdot20\cdot10^{-3}-j\frac{1}{314\cdot500\cdot10^{-6}}=$$

$$=15+j6.28-j6.37=\sqrt{15^2+(6.28-6.37)^2}e^{jarctg(6.28-6.37)/15}=15e^{j0^o}\Omega=15\Omega;$$

$$\underline{Z}_3=Z_3e^{j\psi_{Z3}}=R_3-j\frac{1}{\omega C_3}=20-j\frac{1}{314\cdot400\cdot10^{-6}}=20-j7.96=21.5e^{j21^o}\Omega.$$

We transform each term in the equation from time domain to phasor domain.

2.1. The Kirchhoff's laws method. We have three independent equations on basis of KCL and KVL for the scheme 2.1.b

$$\left.\begin{aligned}
&\underline{I}_{m1}-\underline{I}_{m2}-\underline{I}_{m3}=0;\\
&\underline{Z}_1\underline{I}_{m1}+\underline{Z}_2\underline{I}_{m2}=\underline{E}_{m1};\\
&-\underline{Z}_2\underline{I}_{m2}+\underline{Z}_3\underline{I}_{m3}=-\underline{E}_{m3},
\end{aligned}\right\}$$

Solve the linear system of *3* equations in *3* unknowns $\underline{I}_1,\underline{I}_2,\underline{I}_3$

$$\left.\begin{aligned} \underline{I}_{m1} &= 0.713 - 0.257j = 0.758e^{-j19.8^o}\ A; \\ \underline{I}_{m2} &= 0.0968 + 0.25j = 0.268e^{j68.8^o}\ A; \\ \underline{I}_{m3} &= 0.616 - 0.507j = 0.798e^{-j39.5^o}\ A. \end{aligned}\right\}$$

2.2. The contours currents analysis method.

According to this method each independent circuit (mesh) is flowed around by own contour's current. Equations are drawn up for the contour's currents, which are considered to be as unknown. Once they are determined by currents through the branches of contour's currents. Derivation of basic relations for this method will be on the scheme Fig.2.1.b, which contains two independent contours currents. Each path attribute to the current contour: $\underline{I}_{11}$ and $\underline{I}_{22}$. All contour currents are sent "clockwise." For each of the contours we will make the equation of KVL. In doing so, it should be noted that the general contours of adjacent branches (branch with the impedance $\underline{Z}_2$) are flowed around by differential currents, respectively ($\underline{I}_{11}$ - $\underline{I}_{22}$). The direction of positive bypass circuits in the recording of equations will also be on the "clockwise"

$$\left.\begin{aligned} \underline{Z}_{11}\underline{I}_{m11} - \underline{Z}_{12}\underline{I}_{m22} &= \underline{E}_{m11}; \\ -\underline{Z}_{21}\underline{I}_{m22} + \underline{Z}_{22}\underline{I}_{m22} &= \underline{E}_{m22}, \end{aligned}\right\}$$

where $\underline{Z}_{11} = \underline{Z}_1 + \underline{Z}_2 = 25 + 3.14j, \Omega,$ $\underline{Z}_{22} = \underline{Z}_2 + \underline{Z}_3 = 35 - 7.96j, \Omega$ – own contour impedances for the first and second contours, agreeably; $\underline{Z}_{12} = \underline{Z}_{21} = \underline{Z}_2 = 15, \Omega$ – interfacing contour impedance for the first and second contour; $\underline{E}_{m11} = \underline{E}_{m1} = 9.39 + 3.42j, V, \underline{E}_{m22} = -\underline{E}_{m3} = 6.84 - 18.79, V$ – own contour EMF for the first and second contours, agreeably.

Solve the linear system of *2* equations in *2* unknowns $\underline{I}_{11}, \underline{I}_{22}$

$$\left.\begin{aligned} \underline{I}_{m11} &= 0.713 - 0.257j = 0.758e^{-j19.8^o}\ A; \\ \underline{I}_{m22} &= 0.616 - 0.507j = 0.798e^{-j39.5^o}\ A. \end{aligned}\right\}$$

Having found the contour currents $\underline{I}_{11}$ and $\underline{I}_{22}$ we calculate branches currents. The sought currents are

$$\underline{I}_{m1} = \underline{I}_{m1} = 0.713 - 0.257j = 0.758e^{-j19.8^o}\ A;$$
$$\underline{I}_{m2} = \underline{I}_{m11} - \underline{I}_{m22} = 0.713 - 0.257j - 0.616 + 0.507j = 0.0968 + 0.25j;$$
$$\underline{I}_{m3} = \underline{I}_{m22} = 0.616 - 0.507j = 0.798e^{-j39.5^o}\ A.$$

2.3. The nodal potentials analysis method.

The method of nodal potentials as unknowns is considered potential nodes, which are defined by the first stage of the calculation. Then by the found values of potentials we calculate the currents in the branches. The conclusion of the method we will put for the scheme of Figure 2.18.b. After KCL we can make independent equations (*q-1*)=1, where *q=2* – the number of nodes. We accept the potential of one of the nodes being equal to zero, i.e. conditionally this node is earthed, since any one point of the circuits can be grounded without current distribution changes in the scheme. A potential of the node number 2 is adopted as zero in the scheme of Fig. 2.1.b. We shall write the equation after KCL for node 1:

$$V_{m1}\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)=\frac{E_{m1}}{Z_1}+\frac{E_{m3}}{Z_3}.$$

$$V_{m1}\left(\frac{1}{10+3.14j}+\frac{1}{15}+\frac{1}{20-7.96j}\right)=\frac{9.39+3.42j}{10+3.14j}+\frac{-6.84+18.79j}{20-7.96j}$$

Solve the linear system of one of the equations with one unknown $\underline{V}_{m1}=4.02e^{68.8^o}=1.452+3.747j,V$.

After calculated the node voltage we should find the currents in the branches of scheme:

$$\underline{I}_{m1}=\frac{\underline{V}_{m1}-\underline{E}_{m1}}{\underline{Z}_1}=\frac{(1.452+3.747j)-(9.39+3.42j)}{10+3.14j}=$$

$$=0.713-0.257j=0.758e^{-j19.8^o}\ A;$$

$$\underline{I}_{m2}=\frac{\underline{V}_{m1}}{\underline{Z}_2}=\frac{1.452+3.747j}{15}=0.0968+0.2498j=0.268e^{j68.8^o}\ ,A;$$

$$\underline{I}_{m3}=\frac{\underline{V}_{m1}-\underline{E}_{m3}}{\underline{Z}_3}=\frac{(1.452+3.747j)-(-6.84+18.79j)}{20-7.96j}=$$

$$=0.616-0.507j=0.798e^{-j39.5^o}\ A.$$

2.4. The principle of superposition (The method of overlays)

This method is applicable for the calculation of linear electric circuits, i.e. such circuits, which contain elements with linear volt-ampere characteristics. According to this method, if the circuit contains *n* sources of EMF, the current in a *k*-th branch is the algebraic sum of currents generated in the branches from each of the EMF separately.

The circuit calculation according to the method of superposition is as follows: partial currents are calculated for each energy source, having deleted the rest of energy sources out of the scheme, but leaving in the scheme of the internal impedance of sources, and then find the currents in the branches by the algebraic sum of the partial currents.

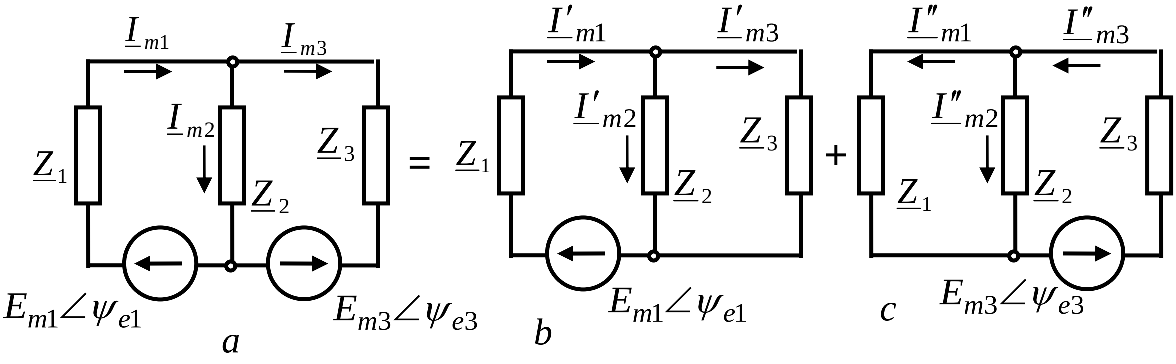


Fig.2.2

This method is considered by the example of the calculation scheme of Fig. 2.2.a. Let us condition that $E_{m1} > E_{m2}$. To calculate the currents in the branches of the scheme (Fig. 2.2.a) by the method of superposition we will present two numerical schemes described in Fig. 2.2.b,c.

Find the currents for the circuit Fig. 2.2.b:

$$\underline{I}'_{m1} = \frac{\underline{E}_{m1}}{\underline{Z}_1 + \dfrac{\underline{Z}_2\underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3}} = \frac{\underline{E}_{m1}(\underline{Z}_2 + \underline{Z}_3)}{\underline{Z}_1(\underline{Z}_2 + \underline{Z}_3) + \underline{Z}_2\underline{Z}_3}$$

$$= \frac{(9.39 + 3.42j)(35 - 7.96j)}{(10 + 3.14j)(35 - 7.96j) + 15(20 - 7.96j)} = \frac{355.87 + 44.95j}{374.99 + 30.3j + 300 - 119.4j} =$$

$$= 0.5 + 0.14j = 0.53e^{j14.7^o}, A;$$

$$\underline{I}'_{m3} = \underline{I}'_{m1}\frac{\underline{Z}_2}{\underline{Z}_2 + \underline{Z}_3} = \frac{\underline{E}_{m1}\underline{Z}_2}{\underline{Z}_1(\underline{Z}_2 + \underline{Z}_3) + \underline{Z}_2\underline{Z}_3} =$$

$$= \frac{(9.39 + 3.42j)15}{(10 + 3.14j)(35 - 7.96j) + 15(20 - 7.96j)} = \frac{140.85 + 51.3j}{374.99 + 30.3j + 300 - 119.4j} =$$

$$= 0.195 + 0.102j = 0.22e^{j14.7^o}, A;$$

$$\underline{I}'_{m2} = \underline{I}'_{m1}\frac{\underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3} = \frac{\underline{E}_{m1}\underline{Z}_3}{\underline{Z}_1(\underline{Z}_2 + \underline{Z}_3) + \underline{Z}_2\underline{Z}_3} = \underline{I}'_{m1} - \underline{I}'_{m3} =$$

$$= 0.5 + 0.14j - (0.195 + 0.102j) = 0.305 + 0.038j = 0.307e^{j7^o}, A.$$

Similarly, we define the currents in the scheme of Fig. 2.2.c

$$\underline{I}''_{m3} = \frac{\underline{E}_{m3}}{\underline{Z}_3 + \dfrac{\underline{Z}_2\underline{Z}_1}{\underline{Z}_2 + \underline{Z}_1}} = \frac{\underline{E}_{m3}(\underline{Z}_2 + \underline{Z}_1)}{\underline{Z}_3(\underline{Z}_2 + \underline{Z}_1) + \underline{Z}_2\underline{Z}_1} =$$

$$= \frac{(-6.84 + 18.79j)(25 + 3.14j)}{(20 - 7.96j)(25 + 3.14j) + 15(10 + 3.14j)} = -0.421 + 0.609j = 0.74e^{j124.7^o}, A;$$

$$\underline{I}''_{m1} = \underline{I}''_{m3}\frac{\underline{Z}_2}{\underline{Z}_2+\underline{Z}_1} = \frac{\underline{E}_{m3}\underline{Z}_2}{\underline{Z}_3(\underline{Z}_2+\underline{Z}_1)+\underline{Z}_2\underline{Z}_1} =$$

$$= \frac{(-6.84+18.79j)15}{(20-7.96j)(25+3.14j)+15(10+3.14j)} = 0.213-0.397j = 0.45e^{-j61.8^o}, A;$$

$$\underline{I}''_{m2} = \underline{I}''_{m3}\frac{\underline{Z}_1}{\underline{Z}_2+\underline{Z}_1} = \frac{\underline{E}_{m1}\underline{Z}_1}{\underline{Z}_3(\underline{Z}_2+\underline{Z}_1)+\underline{Z}_2\underline{Z}_1} = \underline{I}''_{m3} - \underline{I}''_{m1} =$$

$$= -0.421+0.609j-(0.213-0.397j) = -0.2082+0.2118j = 0.297e^{j134.5^o}, A;$$

To determine the currents in the branches we should algebraically sum the partial currents in the positive direction of currents in the scheme Fig.1.2.a:

$$\underline{I}_{m1} = \underline{I}'_{m1} - \underline{I}''_{m1} = 0.5+0.14j-(0.213-0.397j) =$$

$$= 0.713-0.257j = 0.758e^{-j19.8^o} A;$$

$$\underline{I}_{m2} = \underline{I}'_{m2} + \underline{I}''_{m2} = 0.305+0.038j+(-0.2082+0.2118j) =$$

$$= 0.0968+0.2498j = 0.268e^{j68.8^o}, A;$$

$$\underline{I}_{m3} = \underline{I}'_{m3} - \underline{I}''_{m3} = 0.195+0.102j-(-0.421+0.609j) =$$

$$= 0.616-0.507j = 0.798e^{-j39.5^o} A.$$

Converting this to the time domain,

$$i_1(t) = \mathrm{Im}(\underline{I}_{m1}) = \mathrm{Im}(I_{m1}e^{j\psi_{i1}}) = I_{m1}\sin(\omega t+\psi_{i1}) = 0.758\sin(314t-19.8^o), A;$$

$$i_2(t) = \mathrm{Im}(\underline{I}_{m2}) = \mathrm{Im}(I_{m2}\angle\psi_{i2}) = I_{m2}\sin(\omega t+\psi_{i2}) = 0.268\sin(314t+68.8^o), A;$$

$$i_3(t) = \mathrm{Im}(\underline{I}_{m3}) = \mathrm{Im}(I_{m3}e^{j\psi_{i3}}) = \mathrm{Im}(I_{m3}\angle\psi_{i3}) =$$

$$= I_{m3}\sin(\omega t+\psi_{i3}) = 0.798\sin(314t-39.5^0), A.$$

2.5. The method of the equivalent generator.

Whatever active one-port scheme may be replaced in accordance with the method of equivalent generator: EMF equivalent generator is equal to voltage of open circuit, and internal resistance equivalent generator is the input resistance of one-port scheme.

Application of the idea of the equivalent generator method for the calculating parameters of the single third branch scheme presented in Fig. 2.3, a.

We define the current in the impedance $\underline{Z}_2$ at the regime of open circuit. We disconnect the branch with impedance $\underline{Z}_3$ (Fig. 2.3.b) and find the voltage of open circuit:

$$\underline{V}_{m1} = \underline{V}_{m2} + \underline{I}_{moc}\underline{Z}_2 - \underline{E}_{m3} = \underline{V}_{m2} + \frac{\underline{E}_{m1}\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} - \underline{E}_{m3}$$

or

$$\underline{E}_{moc} = -\underline{U}_{moc} = \underline{V}_{m1} - \underline{V}_{m2} = \underline{I}_{moc}\underline{Z}_2 - \underline{E}_{m3} = \frac{\underline{E}_{m1}\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} - \underline{E}_{m3} =$$

$$= \frac{(9.39 + 3.42j)15}{35 + 3.14j} + 6.84 - 1879j = -0.862 - 6.53j = 6.59e^{-j97.5^o}\ ,V.$$

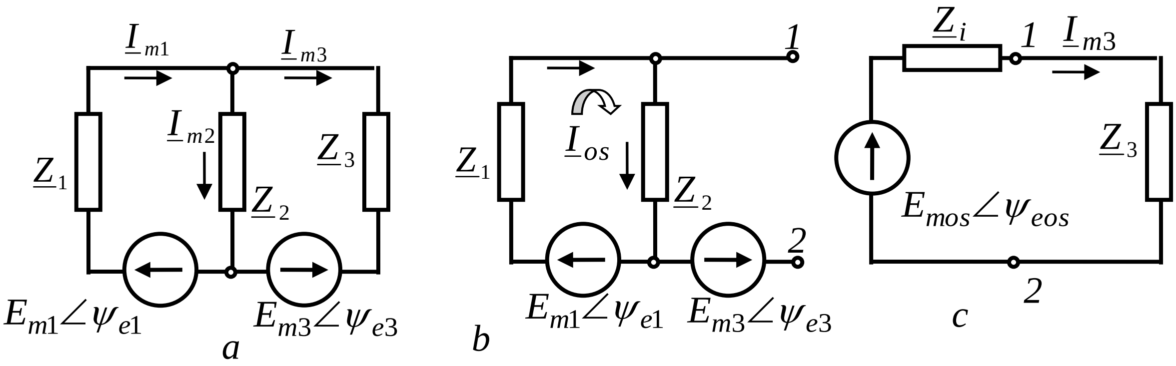


Fig.2.3

The input impedance between the clamps 1-2 is determined by the regime short circuit (Fig. 2.3.b). The points 1 and 2 are connected in a short circuit. Therefore

$$\underline{Z}_i = \frac{\underline{Z}_1\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} = \frac{15(10 + 3.14j)}{15 + 10 + 3.14j} = 4.37 + 0.95j = 4.47e^{j12.3^o}\ \Omega.$$

Finally the current in the third branch is

$$\underline{I}_{m3} = \frac{\underline{E}_{moc}}{\underline{Z}_3 + \underline{Z}_i} = \frac{-0.862 - 6.53j}{20 - 7.96j + 4.37 + 0.95j} = 0.616 - 0.507j = 0.798e^{-j39.5^o}\ A..$$

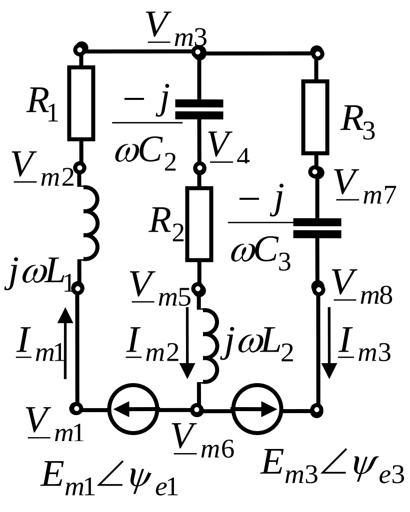


Fig.2.4

2.6. Analysis of the AC scheme with building topographical and phasor diagrams.

For the scheme Fig.2.4 using any analysis of methods of the AC circuits we obtain branches current

$$\left.\begin{aligned} \underline{I}_{m1} &= 0.713 - 0.257j = 0.758e^{-j19.8^o}\ A; \\ \underline{I}_{m2} &= 0.0968 + 0.25j = 0.268e^{j68.8^o}\ A; \\ \underline{I}_{m3} &= 0.616 - 0.507j = 0.798e^{-j39.5^o}\ A. \end{aligned}\right\}$$

Circuits calculated on basis of the values data

$$\underline{E}_{m1} = E_{m1}e^{j\psi_{e1}} = 10e^{j20^o}V = 9.39 + j3.42V;$$

$$\underline{E}_{m3} = E_{m3}e^{j\psi_{e3}} = 20e^{j110^o}V = -6.84 + 18.79V;$$

$$\underline{Z}_1 = Z_1e^{j\psi_{Z1}} = R_1 + j\omega L_1 = 10 + j314\cdot 10\cdot 10^{-3} =$$

$$= \sqrt{10 + 3.14}e^{jarctg(3.13/10)} = 10.48e^{j17.4^o}\Omega = 10 + 3.14j\Omega;$$

$$\underline{Z}_2 = Z_2e^{j\psi_{Z2}} = R_2 + j\omega L_2 - j\frac{1}{\omega C_2} = 15 + j314\cdot 20\cdot 10^{-3} - j\frac{1}{314\cdot 500\cdot 10^{-6}} =$$

$$= 15 + j6.28 - j6.37 = \sqrt{15^2 + (6.28 - 6.37)^2}e^{jarctg(6.28-6.37)/15} = 15e^{j0^o}\Omega = 15\Omega;$$

$$\underline{Z}_3 = Z_3e^{j\psi_{Z3}} = R_3 - j\frac{1}{\omega C_3} = 20 - j\frac{1}{314\cdot 400\cdot 10^{-6}} = 20 - 7.96j = 21.5e^{j21^o}\Omega.$$

Topographical diagram shows potential distribution in electrical circuit.

One of points of the scheme is accepted to be equal to the zero (the point grounding of circuit)

$$\underline{V}_{m1} = 0.$$

Regarding chosen point we find all other potential points of the scheme

$$\underline{V}_{m2} = \underline{V}_{m1} + j\omega L_1\underline{I}_{m1} = \underline{V}_{m1} + jx_{L1}\underline{I}_{m1} = \underline{V}_{m1} + \underline{U}_{L1} =$$

$$= \underline{V}_{m1} + j3.14\cdot 0.758e^{-j19.8^o} = \underline{V}_{m1} + 2.38e^{j70.2^o},V;$$

$$\underline{V}_{m3} = \underline{V}_{m2} + R_1\underline{I}_{m1} = \underline{V}_{m1} + \underline{U}_{R1} = \underline{V}_{m1} + 10\cdot 0.758e^{-j19.8^o} =$$

$$= \underline{V}_{m1} + 7.58e^{-j19.8^o},V;$$

$$\underline{V}_{m4} = \underline{V}_{m3} - j/\omega C_2\underline{I}_{m2} = \underline{V}_{m1} - jx_{C2}\underline{I}_{m2} = \underline{V}_{m1} + \underline{U}_{C2} =$$

$$= \underline{V}_{m1} - j6.37\cdot 0.268e^{j68.8^o} = \underline{V}_{m1} + 1.707e^{-j21.2^o},V;$$

$$\underline{V}_{m5} = \underline{V}_{m4} + R_2\underline{I}_{m2} = \underline{V}_{m4} + \underline{U}_{R2} =$$

$$= \underline{V}_{m4} + 15\cdot 0.268e^{j68.8^o} = \underline{V}_{m4} + 4.02e^{j68.8^o},V;$$

$$\underline{V}_{m6} = \underline{V}_{m5} + j\omega L_2\underline{I}_{m2} = \underline{V}_{m2} + jx_{L2}\underline{I}_{m2} = \underline{V}_{m5} + \underline{U}_{L2} =$$

$$= \underline{V}_{m5} + j6.28\cdot 0.268e^{j68.8^o} = \underline{V}_{m5} + 1.68e^{j158.8^o},V;$$

$$\underline{V}_{m1} = \underline{V}_{m6} + \underline{E}_{m1} = \underline{V}_{m6} + 10e^{j20^o}V = 0;$$

$$\underline{V}_{m7} = \underline{V}_{m3} + R_3\underline{I}_{m3} = \underline{V}_{m3} + \underline{U}_{R3} =$$

$$= \underline{V}_{m3} + 20\cdot 0.798e^{-j39.5^o} = \underline{V}_{m3} + 15.96e^{-j39.5^o},V;$$

$$\underline{V}_{m8} = \underline{V}_{m7} - j/\omega C_3\underline{I}_{m3} = \underline{V}_{m7} - jx_{C3}\underline{I}_{m3} = \underline{V}_{m7} + \underline{U}_{C3} =$$

$$= \underline{V}_{m7} - 7.96j\cdot 0.798e^{-j39.5^o} = \underline{V}_{m7} + 6.35e^{j50.5^o},V;$$

$$\underline{V}_{m6} = \underline{V}_{m8} - \underline{E}_{m3} = \underline{V}_{m8} - 20e^{j110^{o}}, V.$$

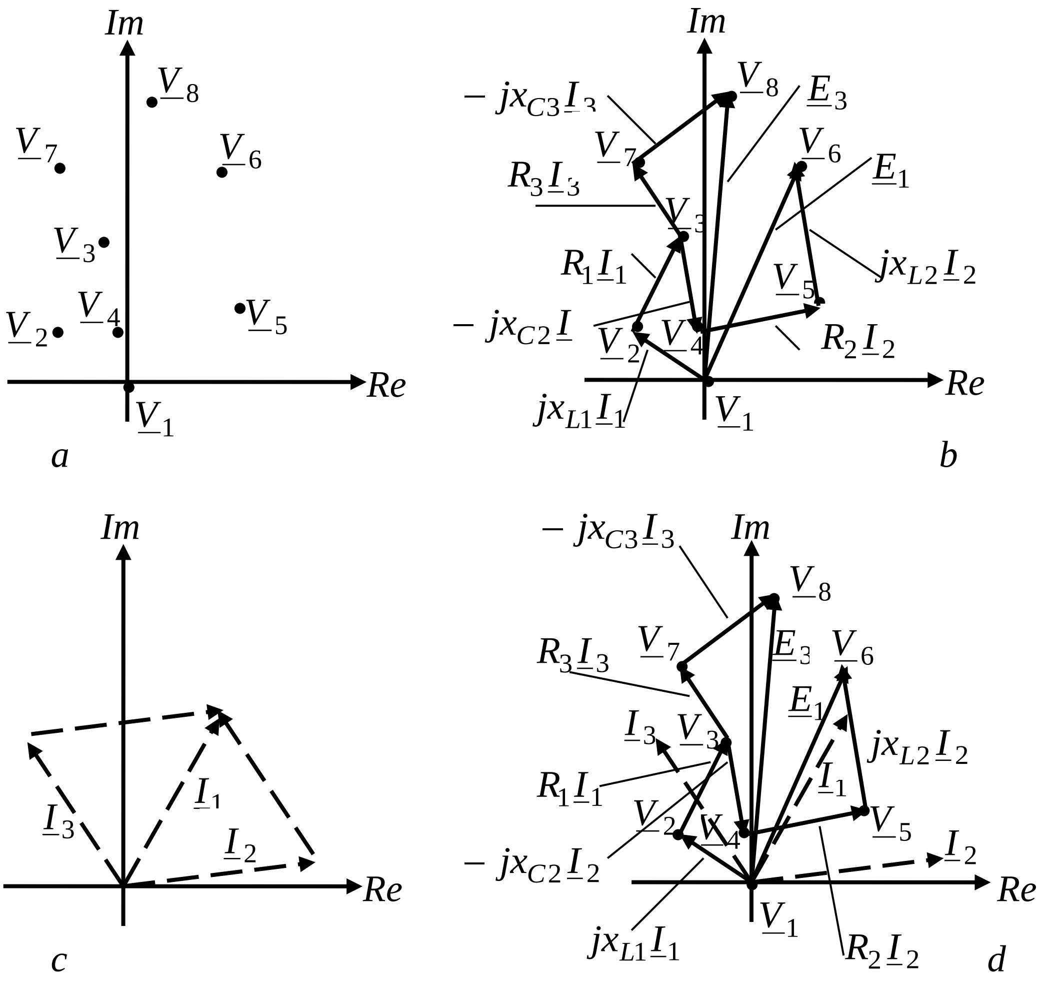


Fig.2.22

Topographical diagram for the scheme Fig.2.21is built in the Fig.2.22. Between points of topographical diagram there are drops of voltages on the scheme elements.

Phasor diagram voltages are obtained from topographical diagram by means of connecting potential points by voltage vectors, Fig.2.22.b.

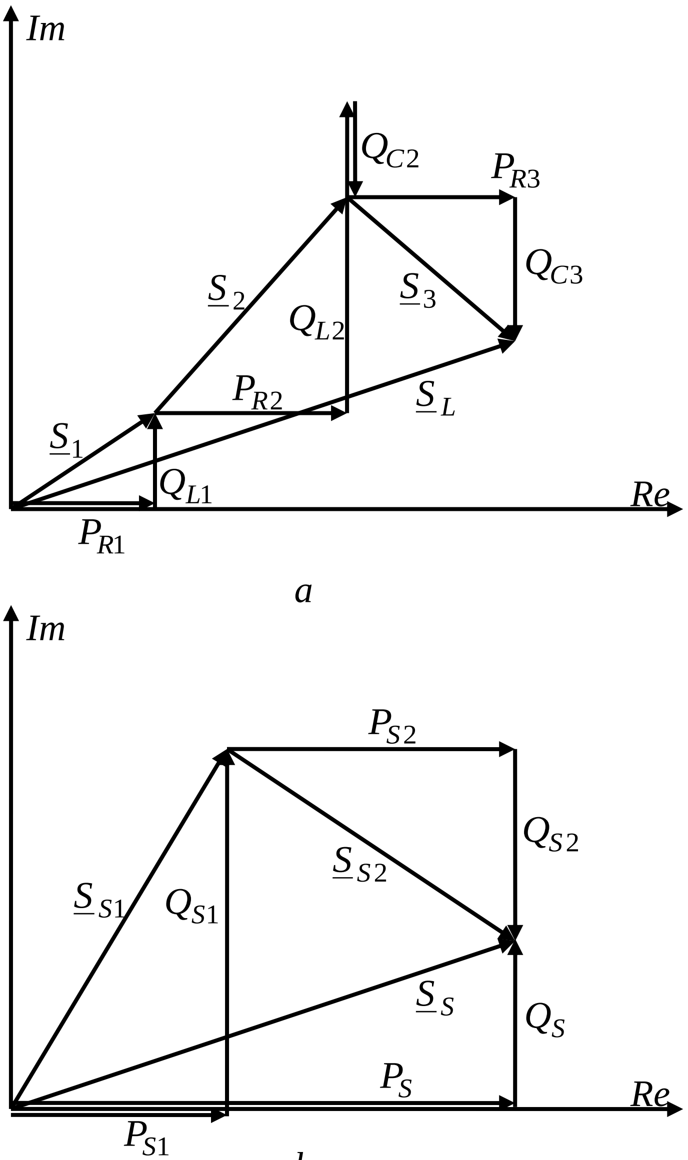


Fig.2.23

A Phasor diagram current is built in Fig.2.22.c. Overlapped Phasor diagrams of the voltages and currents are in Fig.2.22.d.

2.7. AC scheme power analysis (Fig.2.21)

Loads (Fig.2.23.a)

$$P_L = P_{R1} + P_{R2} + P_{R3} = R_1 I_1^2 + R_2 I_2^2 + R_3 I_3^2 = 10 \cdot 0.758^2 + 15 \cdot 0.268^2 + 20 \cdot 0.798^2 =$$
$$= 5.75 + 1.07 + 12.73 = 19.55 W;$$

$$Q_L = Q_{L1} + Q_{L2} - Q_{C2} - Q_{C3} = x_{L1} I_1^2 + x_{L2} I_2^2 - x_{C3} I_3^2 =$$
$$= 3.14 \cdot 0.758^2 + (6.28 - 6.37) \cdot 0.268^2 - 7.96 \cdot 0.798^2 =$$
$$= 1.8 - 0.006 - 5.07 = -3.276 \, \text{var};$$

$$\underline{S}_L = P_L \pm jQ_L = S_L \angle tg^{-1}\left(P_L / Q_L\right) = \sqrt{P_L^2 + Q_L^2} e^{j\varphi} = S_L e^{j\varphi} =$$
$$= 19.55 - 3.276 j = 19.82 e^{-j9.5^o} VA.$$

$\cos\varphi = P_L / S_L$.

Sources (Fig.2.23.b)

$$\underline{S}_1 = \underline{E}_1 \overset{*}{\underline{I}}_1 = 10e^{j20^o}\,0.758e^{j19.8^o} = 5.82 + 4.85j = P_{S1} \pm jQ_{S1};$$

$$\underline{S}_3 = \underline{E}_3 \overset{*}{\underline{I}}_3 = 20e^{j110^o}\,0.798e^{39.5^o} = 13.75 - 8.1j = P_{S3} \pm jQ_{S3};$$

$$\underline{S}_S = \underline{S}_1 + \underline{S}_3 = \underline{E}_1 \overset{*}{\underline{I}}_1 + \underline{E}_3 \overset{*}{\underline{I}}_3 = P_{S1} \pm jQ_{S1} + P_{S3} \pm jQ_{S3} = P_S \pm jQ_S =$$

$$= 5.82 + 4.85j + 13.75 - 8.1j = 19.57 - 3.25j = 19.83e^{-j9.4^o}\,VA.$$

$\cos\varphi = P_S / S_S = 19.57/19.83 = 0.98.$

Power balance in electric circuit (Fig.2.21)

$$P_S = P_{S1} + P_{S3} = P_{R1} + P_{R2} + P_{R3} = P_L;$$

$$\pm Q_S = \pm Q_{S1} \pm Q_{S3} = Q_{L1} + Q_{L2} - Q_{C2} - Q_{C3} = \pm Q_L;$$

$$\underline{S}_S = P_S \pm jQ_S = P_L \pm jQ_L = \underline{S}_L.$$

2.4. Instantaneous values and curve of changing sinusoidal quantity.

The instantaneous values of sinusoidal sizes are set by equations of sinusoids, that are identical for any electric value. For example:

$$e = E_m \sin(\omega t + \psi_e)$$
$$u = U_m \sin(\omega t + \psi_u)$$
$$i = I_m \sin(\omega t + \psi_i)$$

The law of sinusoidals change sizes can be set graphicly. For example fig. 1 presents the scheme over of e(t), made after the first equation. For calculations it is comfortably to replace sinusoidal function with a vector, located on a complex area (fig.2). A vector is made from the beginning of axes of coordinates. Its length is proportional to the module, and orientation in relation to the axis of material numbers "+1" is determined by an initial phase ψ.

The image of sinusoidal value allows setting a vector by its coordinates of two points: the first one is the beginning of coordinates; the second one is of the vector with the coordinates of "a" (for the axes of material numbers) and "b" (for the axes of imaginary numbers). The geometrical sum of these coordinates determines the position of the end of the vector on surface, and is written down in the following way:

$$\dot{E}_m = a + jb$$

And is the imaginary which is used in further calculations. Such form of record of imaginary is called algebraic.

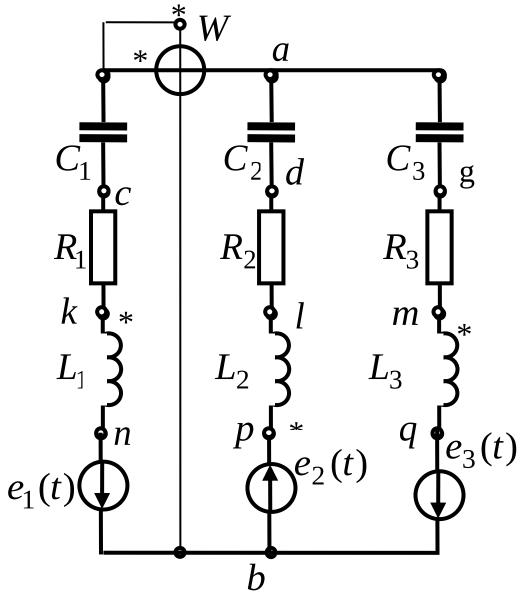


Fig.2.21